COMPETING FOR LOYALTY: THE DYNAMICS OF RALLYING SUPPORT

MATIAS IARYCZOWER AND SANTIAGO OLIVEROS

Abstract. We consider a class of dynamic collective action problems in which either a single principal or two competing principals vie for the support of members of a group. We focus on the dynamic problem that emerges when agents negotiate and commit their support to principals sequentially. We show that competition reduces agents' welfare with public goods, and increases agents' welfare with public bads. Alternatively, competition reduces agents' welfare if and only if there are positive externalities on uncommitted agents. We apply the model to study corporate takeovers, vote buying and exclusive deals. JEL codes: D70, D72, C78.

keywords: contracting externalities, political economy, corporate takeovers, vote buying.

Date: June 30, 2016.

We thank David Ahn, Vincent Anesi, Juliana Bambaci, Antonio Cabrales, Ernesto Dal Bo, Marcelo Fernandez, Faruk Gul, Alessandro Pavan, Ariel Rubinstein and seminar participants at UC Berkeley, U Bocconi, Bristol, Caltech, EUI, Glasgow, Harvard, Northwestern, Princeton, Queen Mary, Rice, St. Andrews, and Surrey for helpful suggestions.

Matias Iaryczower: Department of Politics, Princeton University, Princeton, NJ 08544, email: miaryc@princeton.edu; Santiago Oliveros: Department of Economics, University of Essex, Wivenhoe Park, Essex, CO4 3SQ, e-mail: soliveb@essex.ac.uk.

1. Introduction

Collective action problems can make groups weak and ineffective. This is particularly problematic when an external principal can exploit the incentives of individual members to free ride on each other, leading the group to inefficient or inferior outcomes.

This free-riding problem appears in major applications throughout economics, including public economics (public good provision), political economy (vote buying), and industrial organization (exclusive deals). The classic reference is Grossman and Hart (1980), on corporate takeovers. Grossman and Hart show that externalities across shareholders can prevent takeovers that add value to the company. The idea is that since shareholders that do not sell can capture the increase in value brought by the raider, no shareholder will tender his shares at a price that would allow the raider to profit from the takeover.

The problem is particularly severe when the principal contracts with agents sequentially (Rasmusen et al. (1991), Segal and Whinston (2000), Genicot and Ray (2006)), as it is often the case in practice in political endorsements, vote buying, and exclusive deals.[1] But what if two incumbent firms were to compete for exclusive contracts, or two lobbies were to compete for committee members' votes, or if the incumbent management were to fight a raider's attempt to takeover the firm? Would agents benefit from competition between opposing principals in these contexts?

We address this question within a simple dynamic model in which either a single principal or two competing principals vie for the support of members of a group. Agents choose sequentially between two options, and in so doing interact with principals who attempt to influence their decisions. When enough agents chose one side, that side wins.[2]

Our main result is that competition reduces agents' welfare when the principal's policy provides value to (non-contracting) agents relative to the status quo. This is what we call a *public good* setting. In fact, when agents face two equally good alternatives, agents are always better off when facing a single principal than two competing principals. In general, with public goods, agents are always better off with a monopoly of the best alternative than with competition, even if the difference in the value of the two alternatives

[1]Sequential contracting can arise for various reasons. A common feature of the situations in which sequential contracting is pervasive is the irreversibility of agents' actions, which is induced by contracts, reputational losses, and relation specific investments in different contexts. In this paper, we take sequential contracting as given and explore its consequences.

[2]Prat and Rustichini (2003) also study contracting with multiple principals and multiple agents, but in a static setting with private values and no externalities, and focus on efficiency concerns.

is arbitrarily small. With *public bads*, instead – when the principal's policy is inferior to the status quo – the results reverse and competition is indeed beneficial to agents.

The superiority of monopoly to competition depends crucially on the nature of the free-riding effect. In a public good setting – as it is the case with an efficient raid – agents benefit from not trading with the principal. In this context, free-riding opportunities give agents bargaining power vis-à-vis the principal. Competition harms agents because it reduces the value of free-riding, and thus diminishes their bargaining power.

In the main part of the paper we present a stripped-down model that isolates the key aspects of the problem. In this benchmark model, followers' payoffs in terminal nodes are unaffected by whether they supported the winner, the loser, or remained uncommitted. An indirect consequence of this simplifying assumption is that transfers from principals to agents are positive in the public bad model, but negative with public goods, a feature that can be unappealing in some applications. In Section 4.2 we consider an extended version of the model where we allow payoffs to depend on whether each follower backed a winning or losing candidate, and apply the model to study corporate takeovers, vote buying, and exclusive deals. We show that the direction of the cash transfer is unrelated to whether the decision involves a public good or bad, and hence on whether competition is hurtful or beneficial for followers.

The expanded model shows that the key consideration for our result is whether completion of the project implies a positive or negative externality on uncommitted followers.[3] With positive externalities on uncommitted followers, meeting the principal is bad news for agents, and agents have an incentive to free ride. In the benchmark model this is only due to cash transfers, but the expanded model shows that this happens even when cash transfers go in the opposite direction. The key point is that competition reduces agents' welfare when there are positive externalities on uncommitted followers precisely because in this case competition reduces agents' free riding opportunities.

Our paper contributes primarily to the literature on *contracting with externalities*. These papers explore problems in which a single principal contracts with a group of agents in the presence of externalities among agents.[4] The most closely related papers are Rasmusen et al. (1991), Segal and Whinston (2000) and Genicot and Ray (2006), which study sequential

[3]The importance of positive and negative externalities, as well as increasing or decreasing externalities among agents in contracting models was first highlighted in a static setting by Segal (1999). The focus of this paper is on how the type of externalities among agents affect efficiency. See also Segal (2003).

[4]See Segal (1999) for a model of contracting with externalities that unifies various applications.

contracting.[5] These papers show that when agents impose negative externalities on each other by contracting with the principal, sequential contracting exacerbates the free-rider problem among agents.[6] The literature on contracting with externalities is closely related to our monopolistic model (Section 3), where the externality is due to the fact that agents make a collective decision. The key innovation of our paper is to extend the model of sequential contracting with externalities to a competitive setup and –primarily – to compare the welfare of agents under monopoly and competition.[7] In fact, as we foreshadowed above, the result that monopoly is preferred to competition with public goods depends crucially on the nature of the free-riding effect introduced by Grossman and Hart (1980), and is linked to the direction of externalities on non-traders emphasized by Segal (1999).

Our paper also contributes to the literature on *vote buying* and interest group influence developed in political economy. In these papers, two lobbyists compete offering transfers to members of a committee to obtain their support in favor or against a bill (see for example Myerson (1993), Groseclose and Snyder (1996)). Our paper is closest to Dekel et al. (2008, 2009). The fundamental innovation here is to introduce sequential vote buying with two principals and strategic agents. Our paper is also related to the models of innovation races pioneered by Harris and Vickers (1985, 1987). However there are important differences. Most directly, while we focus on agents' welfare, these agents are absent in the models of races, which focus on the game among principals. As a result, the free riding problem at the core of our paper is absent in this literature, as is the interaction of principals with strategic, forward-looking agents.[8]

Finally, our conclusion has a point of contact with Olson (2000)'s governance argument that a stationary monopolistic bandit may be better than competition. Both arguments are fundamentally about externalities. Olson's argument is that the monopolistic leader internalizes to a larger extent the effect of her actions on total surplus. In our model,

[5]See also Galasso (2008), Möller (2007), and Rasmusen and Ramseyer (1994).

[6]Segal and Whinston (2000), Segal (2003) and Dal Bo (2007) show that the principal can exploit agents in a static setting if she can make discriminatory offers.

[7]The literature on non-cooperative coalitional bargaining games with externalities considers general bargaining problems with multiple agents (Bloch (1996), Ray and Vohra (1999), Ray and Vohra (2001), Gomes (2005), Gomes and Jehiel (2005)). These papers aim to establish efficiency, existence and uniqueness properties in this class of games.

[8]More indirectly, our paper connects with the literature on competition in public goods (see Bergstrom et al. (1986), Ghosh et al. (2007)), in which individuals simultaneously choose contributions to multiple public goods. In these papers consumers do not interact with principals influencing their decision.

instead, agents prefer monopoly to competition when competition reduces their bargaining power vis a vis the leaders by reducing free riding opportunities.

2. The Model

Two leaders, A and B, compete to gather the support of a majority of agents (followers) in a group of size n. The first leader to obtain the commitment of $q \equiv (n+1)/2$ members wins, and implements her preferred alternative. There is an infinite number of periods, $t = 1, 2, \ldots$. In each period t before a leader wins, any one of the $k(t)$ uncommitted followers at time t meets leader $\ell = A, B$ with probability $\pi_\ell / k(t) > 0$, where $\pi_A + \pi_B = 1$. Say that at the time of the meeting, ℓ needs m_ℓ additional followers to win, and let $\vec{m} \equiv (m_A, m_B)$. In the meeting with an uncommitted follower i, leader ℓ makes a TIOLI offer $p_\ell(\vec{m}) \in \mathbb{R}$ to secure i's support.[9] Follower i can accept or reject ℓ's offer; if he accepts, i commits his support for ℓ and receives $p_\ell(\vec{m})$, if he rejects, i remains uncommitted.

Leader ℓ gets a payoff of $\overline{v}_\ell > 0$ if and when she wins, and $\underline{v}_\ell$ if and when her opponent $j \neq \ell$ wins, $\overline{v}_\ell > \underline{v}_\ell$. In any period before a leader wins, both leaders get a payoff of zero (a normalization). Followers get a payoff w_ℓ if and when leader ℓ wins, and a payoff of zero in each period t before a leader wins. We initially assume that both leaders provide value to the agents; i.e., $w_\ell > 0$ for $\ell = A, B$. We refer to this case as a *public good* case. We then consider the *public bad* case, in which $w_\ell < 0$ for $\ell = A, B$. Leaders and followers have discount factor $\delta \in (0, 1)$.

The solution concept is Markov Perfect equilibrium (MPE). We let $W(\vec{m})$ denote the continuation value of an uncommitted follower in state $\vec{m} \equiv (m_A, m_B)$, $W_{out}(\vec{m})$ denote the continuation value of a committed follower in state $\vec{m}$, and $V_\ell(\vec{m})$ denote the continuation value of leader ℓ in state $\vec{m}$. We also define $\vec{m}^A \equiv (m_A - 1, m_B)$ and $\vec{m}^B \equiv (m_A, m_B - 1)$.

As a benchmark, we also consider the case in which there is only one alternative to the status quo. The model is the same as before, with $\pi_\ell = 0$ for some $\ell = A, B$. We denote the payoffs that the leader and a follower obtain when the leader wins, by v and w. In the public good case, we initially let $w > 0$. Leader and followers obtain a payoff of zero in any period before the leader wins. The state is the number m of additional followers the leader needs to win. The offer in state m is $p(m)$, the value of an uncommitted and committed followers are $w(m)$ and $w_{out}(m)$, and the leader's value $v(m)$.

[9]This assumption is prevalent in the literature on contracting with externalities (see however Galasso (2008)). Giving followers some bargaining power does not change our results.

3. Monopolistic Leadership

We begin by characterizing MPE of the public good model with a single alternative. Here we proceed informally. Consider first the critical follower facing the leader when $m = 1$. Note that since $v, w > 0$, leader and follower would jointly benefit from moving forward, securing the leader's victory. Thus there is a transfer that makes this mutually beneficial, and in any equilibrium there must be a transaction when $m = 1$.

In fact in equilibrium this must be true in all states $m \leq q$: the leader makes an offer in every meeting until she collects a majority of followers, and the uncommitted followers who meet the leader accept these offers. We refer to this as a full trading equilibrium (FTE). To see why this is an equilibrium, suppose there are $m > 1$ followers remaining. Since in the proposed equilibrium there is trade whenever the leader needs to secure the support of $t < m$ additional followers, in m it is also jointly beneficial for leader and follower to move forward, just as in $m = 1$. As a result, here again there is a transfer that makes an agreement mutually beneficial.

Now consider any MPE, and suppose that there is trade whenever the leader needs to secure the support of $t < m$ additional followers. If in state m the leader does not make an offer with positive probability or the follower does not accept the offer with positive probability, leader and follower would obtain a lower combined payoff in state m than in the equilibrium with trade. Hence the gain from moving forward would be higher than in the FTE, and thus positive. It follows that the leader will make an offer, which the follower will accept. We then have

Proposition 3.1. *The unique MPE of the game $\Gamma^s(q)$ is a full trading equilibrium. In this equilibrium, the payoff of a follower is given by*

$$w(q) = \left(\prod_{m=1}^{q} r(m)\right) \delta^q w \quad \textit{for} \quad r(m) \equiv \frac{n + 2m - 3}{n + 2m - 1 - 2\delta}$$

Proposition 3.1 implies that in equilibrium the leader cannot extract all surplus from the followers. The logic is similar to that behind under-provision of a public good. Note that since followers benefit from implementing the alternative to the status quo, the leader actually charges them to move on. By rejecting the offer, a follower can rely on others to pay the bill. This generates an outside option that gives followers bargaining power over the leader. Since the cost of deferring implementation of the proposal decreases with δ, the value of the outside option is increasing in δ, and so is followers' equilibrium payoff. In fact, as δ approaches 1, $r(m) \to 1$ and $w(q) \to w$.[10]

4. Competition and Main Result

Consider next the competitive game $\Gamma^c(q,q)$, with two alternatives to the status quo, A and B, and initial position $\vec{m} = (q,q)$. Recall that we defined $\vec{m}^A \equiv (m_A - 1, m_B)$ and $\vec{m}^B \equiv (m_A, m_B - 1)$. Follower i accepts an offer $p_\ell(\vec{m})$ from ℓ only if $p_\ell(\vec{m}) + \delta W_{out}(\vec{m}^\ell) \geq \delta W(\vec{m})$, and accepts with probability one if this inequality is strict. Thus in equilibrium, if ℓ makes an offer, she offers

$$p^*_\ell(\vec{m}) \equiv -\delta[W_{out}(\vec{m}^\ell) - W(\vec{m})]. \tag{1}$$

The offer by ℓ has to compensate i for what he would obtain if he were to refuse to commit his support for ℓ, taking him – along with the entire group – back to a position $\vec{m}$. Leader ℓ is willing to make this offer if $p^*_\ell(\vec{m}) \leq \delta[V_\ell(\vec{m}^\ell) - V_\ell(\vec{m})]$, or substituting $p^*_\ell(\vec{m})$ from (1), if the surplus for i and ℓ of taking ℓ one step closer to the goal is nonnegative; i.e.,

$$S_\ell(\vec{m}) \equiv [V_\ell(\vec{m}^\ell) - V_\ell(\vec{m})] + [W_{out}(\vec{m}^\ell) - W(\vec{m})] \geq 0. \tag{2}$$

In the natural extension of the full trading equilibrium in the monopolistic case to the competitive game, leaders make relevant offers in all states $\vec{m}$ such that $m_A, m_B \leq q$. Differently than its counterpart in the monopoly game, this strategy profile is not always an equilibrium in this context. With this in mind, in the next proposition we establish two results. First, we prove that the competitive game has a MPE equilibrium. Second, we provide a sufficient condition for the existence of a FTE in the competitive game.

Proposition 4.1. *(i) The competitive game $\Gamma^c(q,q)$ has a MPE; (ii) All else equal, if the leaders' payoff for winning is sufficiently high, the game has a FTE, with the value of an uncommitted follower in state $\vec{m}$ given by*

$$W(\vec{m}) = \sum_{j=A,B} (\delta\pi_j)^{m_j} \left[\sum_{l=0}^{m_{-j}-2} \left(\prod_{k=0}^{m_j-1+l} r\left(m_{-j}+m_j-k-\frac{n+1}{2}\right) \right) \binom{m_j-1+l}{m_j-1} (\delta\pi_{-j})^l \right] w_j, \tag{3}$$

where we have adopted the convention that for any $f(\cdot)$, $\sum_{k=a}^{b} f(k) = 0$ if $b < a$.[11,12]

---

[10]Incidentally, we note that since $w(m)/\delta w(m-1) = r(m) < 1$ for all m, conditional on trading at some point, a follower is better off if approached later in the process.

[11]Note that $W(\vec{m}) \to w_A(m)$ (as characterized in Proposition 3.1) as $\pi_B \to 0$.

[12]Note this equilibrium is inefficient, as the worst alternative wins with positive probability. This result is in line with the literature on non-cooperative coalitional bargaining games with externalities, which

We are now ready to compare followers' equilibrium payoffs under monopoly and competition. We begin by considering FTE of the competitive game with three followers.

Example 4.2. *Suppose $n = 3$. Consider first the state $\vec{m} = (1,1)$, reached after one follower has committed for each leader. From (1), leader $\ell = A, B$ offers the follower $p_\ell(1,1) = \delta[W(1,1) - w_\ell]$, which the follower accepts. Thus*

$$W(1,1) = \pi_A (p_A(1,1) + \delta w_A) + \pi_B (p_B(1,1) + \delta w_B) = \delta W(1,1),$$

so that $W(1,1) = 0$. Because only one follower remains uncommitted there are no free riding possibilities and all surplus is extracted by the leaders. Consider next the state $\vec{m} = (1,2)$. As before, a follower meeting the leader gets a payoff $\delta W(1,2)$, and thus

$$W(1,2) = \frac{1}{2}\delta W(1,2) + \frac{1}{2}(\pi_A \delta w_A + \pi_B \delta W(1,1))$$

Thus $W(1,2) = (\delta\pi_A/(2-\delta))\, w_A > 0 = W(1,1)$. Similarly, $W(2,1) = (\delta\pi_B/(2-\delta))\, w_B$ and then

$$W(2,2) = \left(\frac{2\delta^2}{3-\delta}\frac{1}{2-\delta}\right)\left((\pi_A)^2 w_A + (\pi_B)^2 w_B\right) \tag{4}$$

Now, from the single leader case we have,

$$w(2) = \left(\frac{2\delta^2}{3-\delta}\frac{1}{2-\delta}\right) w$$

Note that the expression in parenthesis is equal to the corresponding expression in (4). Thus, if $w = w_A \geq w_B$, followers prefer a monopoly of A to competition between A and B, for any discount factor $\delta < 1$. Furthermore, when $\Delta w = w_A - w_B$ is small, followers also prefer a monopoly of B to competition if $\pi_A \approx 1/2$. □

The fact that in this example there are only three agents is not important for this result. In fact, a comparison of followers' value in Proposition 4.1 and Proposition 3.1 shows that the example generalizes to FTE with n agents. The example illustrates two important lessons. First, since a monopoly can improve followers' welfare relative to competition even for $\delta \to 1$, the reason why followers are better off under single leadership is not solely explained by delay. Second, because sometimes a monopoly of the worst leader dominates competition, the reason behind the followers' welfare ordering cannot be solely due to the "risk" of ending up collectively selecting the worst choice available.

has shown that inefficiency is the norm in these class of games (Bloch (1996), Ray and Vohra (1999), Ray and Vohra (2001), Gomes (2005), Gomes and Jehiel (2005)).

The key to understand the result is in how the followers' bargaining power changes across the two games. Followers prefer monopoly to competition because competition reduces their free-riding ability, which directly lowers their bargaining power vis a vis the leaders. This is captured in eq. (3), which shows that the source of value is the probability of all paths leading to successful free riding until completion.

To see this intuitively, consider the problem of the *critical* follower in monopoly (the q^{th} member to meet the leader). If the critical follower refuses an offer by the leader, he will be able to free ride on others with probability $(q-1)/q = (n-1)/(n+1)$. This relatively high free-riding ability gives the critical follower considerable bargaining power against the leader and protects him from getting fully expropriated. Now consider the problem of a critical follower under competition (a follower who could give one of the leaders a win). The number of uncommitted followers in this situation is at most as large as in monopoly. In fact, if the battle between the leaders is even until the end, the critical follower could meet the leader in state $\vec{m} = (1,1)$. But here there are no free riding possibilities.

This situation is extreme, but the logic generalizes to other states in which the number of uncommitted followers is lower than in the single alternative case. To see this, consider the competitive and monopolistic games with $n = 5$ and $w_A = w_B = w$. Using Proposition 3.1 and eq. (3), $w(3) > W(3,3)$ boils down to $1 > \sum_{j=A,B} (\pi_j)^3 \left[1 + \frac{3}{2-\delta}(\delta\pi_{-j})\right]$. Letting $\gamma(\vec{m})$ denote the probability of reaching the node $\vec{m}$ from the initial node (q,q) in a competitive equilibrium, we can write this as

$$(1-\delta)\left[\gamma(2,1) + \gamma(1,2) + \gamma(1,1)\right] + \frac{1}{2}\delta\gamma(1,1) > 0.$$

This shows that the difference in value across games comes from those nodes in which the number of uncommitted followers cannot be reached in monopoly.[13]

In the discussion so far we have assumed a FTE in the competitive game. For some parameters, though, equilibrium will require that one of the leaders does not make an offer in some states, or only makes an offer with positive probability. Moreover, in contrast to the monopoly game, we can not guarantee that the equilibrium of the competitive game will be unique. Theorem 4.3 – our main result – shows that the result holds for *any* number of agents n, and *any* MPE.

[13]Both delay and free-riding vanish as $\delta \to 1$, because patience eliminates the source of power that free-riding gives to the followers. However, these are two distinct effects. In fact, delay is not necessary for the result. Note that as $\delta \to 1$ all terms vanish except $\vec{m} = (1,1)$, in which the remaining follower has no opportunity to free-ride.

Theorem 4.3. *Let $W(q,q)$ denote the payoff of an uncommitted follower in a MPE of the game with two alternatives A and B such that $w_A \geq w_B > 0$. Then (i) $W(q,q) < w_A(q)$. Moreover, (ii) there exists $\varepsilon > 0$ such that if $w_A - w_B < \varepsilon$, $W(q,q) < w_B(q)$.*

The proof is by induction and consists of four steps:

(1) In any state $\vec{m}$ there is always an alternative $j \in \{A, B\}$ s.t. followers are better off after moving one step in the direction of j; i.e., $W(\vec{m}) < \max_{j\in\{A,B\}}\{W(\vec{m}^j)\}$.
(2) When a principal j is one step away from winning followers would be strictly better off by removing one of the alternatives from consideration; e.g., $W(m_A, 1) < \max\{w_A(m_A), w_B(1)\}$. (Connects the competitive and monopolistic games.)
(3) Combining steps 1 and 2 we show that when principal j is two steps away from winning followers would be strictly better off removing one of the alternatives from consideration; e.g., $W(m_A, 2) < \max\{w_A(m_A), w_B(2)\}$ for all $m_A \geq 2$.
(4) Induction step for $\vec{m} \geq (3,3)$: if $W(\vec{m}^B) \leq \max\{w_A(m_A), w_B(m_B - 1)\}$ and $W(\vec{m}^A) \leq \max\{w_A(m_A - 1), w_B(m_B)\}$ then $W(\vec{m}) \leq \max\{w_A(m_A), w_B(m_B)\}$.

Theorem 4.3 states that a monopoly of the better alternative is better for followers than competition at the beginning of the game, from a position of symmetry.[14] The sketch of the proof, however, makes clear that the comparison holds in the entire state space. It follows that the presence of an initial advantage would not alter our results.

Corollary 4.4. *Let $W(q_A, q_B)$ denote a follower's payoff in a MPE of the competitive game with initial position (q_A, q_B), $q_A < q_B$. Then $W(q_A, q_B) < \max\{w_A(q_A), w_B(q_B)\}$.*

Corollary 4.4 allows us to extend the benchmark model to arbitrary non-unanimous q-rules. Say B needs to obtain the support of a supermajority $q_B > (n+1)/2$ of members to implement a reform leading to a value of $w_B > 0$ for the followers, while A can block the reform by getting the support of $q_A = n - q_B$ members, leading to $w_A = 0$. This model is formally equivalent to introducing initial advantages, with the exception that $w_A = 0$. Thus Corollary 4.4 applies to this case as stated.[15]

[14]Our statement concerns only followers' welfare. In the special case of $\delta \to 1$, $w_A = w_B = w$, and a FTE in competition, the overall surplus is in fact unchanged and thus the aggregate welfare of the leaders is larger in competition. In general, competition can increase or decrease total welfare.

[15]Because unanimity eliminates free riding opportunities, monopoly under unanimity is worst for followers than competition under simple majority. We can then show that for any submajority rule Q in competition there exists a rule $\hat{q}(Q) \geq Q$ in monopoly such that the value of an uncommitted follower under competition with rule Q is at most (at least) the value of an uncommitted follower under monopoly with rule $\hat{q}(Q) - 1$ (with rule $\hat{q}(Q)$). We thank an anonymous referee for pointing this out.

Part (i) of Theorem 4.3 and Corollary 4.4 focused on the comparison of competition with a monopoly of the best alternative. Part (ii) of the theorem, on the other hand, shows that followers can prefer a monopoly of the *worst* alternative to competition, provided that the two alternatives are sufficiently close to one another. This outcome is striking. In our next result we pursue this further, and provide conditions under which this result can be extended to *any* $w_A, w_B > 0$.

Theorem 4.5. *Let $W(q,q)$ denote the payoff of an uncommitted follower in a FTE of the competitive game with $w_A, w_B > 0$. Then $\exists\ \overline{n}$ s.t. if $q = (n+1)/2 > \overline{n}$, $W(q,q) < \min\{w_A(q), w_B(q)\}$.*

Theorem 4.5 builds on the same core intuition behind our main result. Note that in monopoly, the probability that the critical uncommitted follower will be able to free ride after refusing an offer from the leader is $(n-1)/(n+1)$, which goes to one as $n \to \infty$. On the other hand, the competitive game can turn out to be very close, even in a large group. In this case a critical follower in the competitive game can find himself to be one of only a few uncommitted followers, and as a result have few free-riding opportunities and therefore less bargaining power. This effect trickles down to the beginning of the game, and then even a monopoly of the worst alternative is preferred to competition.

We have repeatedly emphasized the crucial role of free-riding for our result. A natural question then is whether Theorem 4.3 can be generalized to $K > 2$ principals. Intuition suggests that if anything, the free-riding argument should be even stronger in this case.

In order to do this, we need to deal with the complexity induced by the higher dimensionality of the problem. A key object in the two leader problem is the set of states in which one trade exhausts free riding opportunities. With two leaders, this happens at $\vec{m} = (1,2)$ with a trade with B, and at $\vec{m} = (2,1)$ with a trade with A. With more than two leaders there are multiple states in which a single transaction exhausts free riding opportunities, and in any such state typically multiple such transactions. The idea that allows us to deal with this higher dimensionality is to recognize that the states in which a transaction exhausts free riding opportunities are conceptually equivalent to each other, and similarly that the states in which no transaction exhausts free riding opportunities are conceptually equivalent to each other, so that the key at any state is to separate transactions that lead the process to one set or the other set of states. From this point on we can then prove the result with a generalization of the arguments in the proof of Theorem 4.3.

Theorem 4.6. *Let $W^K(\vec{q})$ denote the payoff of an uncommitted follower in a MPE of the game with $K > 2$ alternatives $\ell = 1, 2, \ldots, K$ such that $w_1 > w_2 > \ldots > w_K > 0$, with initial position $\vec{q} = (q_1, q_2, \ldots, q_K)$. Then (i) $W^K(\vec{q}) < \max_\ell\{w_\ell(q_\ell)\}$. Moreover, (ii) if $q_\ell = q \ \forall \ell \in K$, $\exists\, \varepsilon > 0$ s.t. for any $\ell = 1, \ldots, K$, if $w_1 - w_\ell < \varepsilon$, $W^K(\vec{q}) < w_\ell(q)$.*

4.1. **Public Bads.** Up to this point we assumed that the alternatives under consideration are public goods, in the sense that followers prefer the outcome associated with a victory of A or B to the status quo. The logic of the proof of Theorem 4.3, however, suggests that the analysis above applies to *public bads* (i.e., $w_A, w_B < 0$) almost unchanged, reverting the result. Intuitively, since with public bads followers can only cut their loses if they are part of the coalition that supports the leader, a critical agent that refuses the offer with a single alternative will have a much lower chance of being brought back to the table of negotiations than a critical follower in a competitive environment, and as a result can demand a higher transfer in exchange of his support.

In order to apply this result, we first need to establish FTE in monopoly with public bads. The next proposition shows that the FTE is still the unique MPE as long as the leader's win value is sufficiently high, and moreover that any MPE with transactions is a FTE.

Proposition 4.7. *Let $w < 0$. Any MPE is either a FTE or involves no transactions. Moreover, $\exists \overline{v} > 0$ s.t. if $v \geq \overline{v}$, the FTE is the unique MPE of the game.*

Proposition 4.7 says that if the leader's win value is sufficiently large, a leader representing a policy that is arbitrarily bad for followers can win with the support of a majority of the group. This is of course terrible news for those agents who are not compensated. In fact, $w(q)$ goes to the upper bound of zero as $\delta \to 0$, and to the lower bound of $w < 0$ as $\delta \to 1$. The reason for this result is that the leader uses uncommitted followers against each other, as in Genicot and Ray (2006). Upon meeting, both leader and follower know that if the follower rejects the leader's offer, the next follower will accept it. Thus, the follower can only delay the implementation of the public bad for one period, if he forgoes any compensation. To prevent this delay, the leader can offer to compensate the follower *for this differential only* (and not for the full cost that implementation will bring to the follower).

With the previous result, we can establish formally our main conclusion.[16]

[16]Part (ii) of Theorem 4.3 goes as well, as do Corollary 4.4 and Theorem 4.5.

Proposition 4.8. *Let $w_A, w_B < 0$. Let $W(q,q)$ denote the initial value of an uncommitted follower in a MPE of the competitive game, and let $w_k(q)$ denote the initial value of an uncommitted follower in a MPE with trade of the game in which $k = A, B$ is the only principal. Then $W(q,q) \geq \min\{w_A(q), w_B(q)\}$.*

4.2. **Applications.** Up to this point we have studied a stripped-down model in which followers' payoffs in terminal nodes are unaffected by whether they supported the winner, the loser, or remained uncommitted. This model allows us to isolate the key aspects of the problem, but is not well suited for applications, where this distinction is often natural or necessary. In this section we consider an extended version of the model where we distinguish between the *insider* payoff z_ℓ obtained by an individual who gave her support to ℓ when ℓ wins, the *outsider* payoff w_ℓ obtained by an uncommitted follower when ℓ wins, and – in the competitive model – the *rival* payoff y_ℓ obtained by an individual who gave her support to $j \neq \ell$ when ℓ wins, and take the model to applications.

Example 4.9. *[Corporate Takeovers] Grossman and Hart (1980) (GH) analyze a problem in which a company (the raider) acquires shares of a target company to control its board of directors. It is assumed that the raider can improve the value of the company. To capture this, we assume that under the raider's control the value of a share is $w > 0$, and normalize the value of a share under the incumbent management to zero. We distinguish the payoff of a shareholder who does not sell to the raider if the raider wins ($w > 0$) form the payoff of a shareholder who sells to the raider if the raider wins ($z = 0$).*[17] *We then consider introducing competition from a second principal (e.g., the incumbent management)*

Example 4.10. *[Vote Buying] Here we consider a model of sequential vote buying.*[18] *We think of this model as capturing a process of coalition building that occurs in office deals prior to the moment the proposal is on the floor for a vote, at which point allegiances are typically already decided.*[19] *To fix ideas, then, consider a national legislature which is about to vote on a fiscal restraint bill proposed by the executive. We assume that legislators – who understand the dire state of fiscal affairs for the state – privately favor the bill ($w > 0$). Voters, however, oppose it, so that supporting the bill is costly for legislators;*

[17] As in GH and Segal (2003), we assume shareholders are homogeneous. Differently than GH, we suppose that shareholders are fully aware of the effect of their action on the outcome of the raid attempt. Holmstrom and Nalebuff (1992) show that when shareholdings are divisible the free-riding problem does not prevent the takeover process in the GH model.

[18] See Rasmusen and Ramseyer (1994), Genicot and Ray (2006) with a single principal, and Dekel et al. (2008, 2009) with two principals and non-strategic agents.

[19] This is in the spirit of Iaryczower and Oliveros (2016), who study intermediaries in legislative bargaining.

i.e., $z < 0 < w$. *We then introduce a second option. The executive's proposal (say A) aims to reduce the deficit by increasing taxes, while an alternative proposal favored by a powerful lobby (B) seeks to reduce public expenditures. We assume that legislators prefer increasing taxes, so that* $w_A > w_B > 0$.

Example 4.11. *[Exclusive Deals] We consider a problem in which two firms compete for the market of a product with increasing returns to scale signing exclusive contracts with buyers. Our motivating example is the HD optical disc format war between Blu-ray and HD-DVD. This problem relates to the analysis in Rasmusen et al. (1991) and Segal and Whinston (2000), with two key differences. First, we allow both the incumbent and the "challenger" to sign exclusive contracts. We also assume that both firms are initially competing in the market. Thus, ours is a model where firms take actions to induce exit, as opposed to deterring entry. Suppose without loss of generality, that A is the better technology. Then* $y_A \geq y_B$ *and* $z_A = w_A \geq z_B = w_B$. *As is standard in the literature we assume that buyers' prices would increase if one of the incumbents exits, so that* $w_j = z_j < 0$ *for* $j = A, B$.

Consider first a FTE in monopoly.[20] Because in equilibrium the follower is indifferent between accepting or rejecting the offer, the payoff of a follower meeting the leader in state m is $\delta\hat{w}(m)$. As a result, the recursive representation of the value of uncommitted followers is unchanged from the benchmark game (see eq. (8) in the Proof of Proposition 3.1), and thus so is its solution, $\hat{w}(m) = \left(\prod_{k=1}^{m} r(m)\right)\delta^m w$. The value of a committed follower, on the other hand, is now given by $\hat{w}_{out}(m) = \delta^m z \leq 0$ in examples 4.9, 4.10, and 4.11. Substituting,

$$\hat{p}(m) = \delta^{m+1}\left(\left[\prod_{k=1}^{m} r(k)\right] w - z\right) > 0, \tag{5}$$

so that transfers are positive in all three cases even as $w > 0$. In fact, $\hat{p}(m) \to w - z$ for all m as $\delta \to 1$. In the takeover context, this means that as frictions vanish the raider has to fully compensate shareholders, as in GH. For any $\delta < 1$, however, $\hat{p}(m) < w$, and the raider can appropriate some of the surplus it generates.[21] In addition, from (5) we

[20]In Proposition A.9 we show that whenever $z \leq 0 < w$, the FTE is the only robust MPE with trade, and this is the unique MPE for high v. The result for example 4.11 follows directly from Proposition 4.7.
[21]This result is similar to that of Harrington and Prokop (1993), who consider a dynamic version of GH in which the raider can re-approach the shareholders who have not sold (at the posted prices). One should not conclude from this that the leader would always prefer sequential to static contracting or vice versa (See Appendix 7.5).

have $\hat{p}(m-1) - \hat{p}(m) > 0$. Thus, the price curve is decreasing in m, so that agents who transact first obtain a lower compensation.

How would competition affect agents' welfare in these contexts? In the benchmark model we saw that whenever principals add value, competition is detrimental to followers' welfare. But now we have positive transfers. The fact is that our main result is not affected by the direction of transfers. The reason is that in equilibrium followers are indifferent between accepting or rejecting the offer, and the payoff of a follower meeting the leader in state $\vec{m}$ is $\delta\hat{W}(\vec{m})$, independently of whether he accepts or rejects the leader's offer. Because of this, as in monopoly, the recursive representation of the followers value is unchanged from the benchmark model (see eq. (16) in the Proof of Proposition 4.1) and thus so is its solution; i.e., $\hat{W}(\vec{m}) = W(\vec{m})$. This implies that the followers' value only depends on the outsider payoffs w_ℓ, with all the new elements going into the "revised" cash transfer $\hat{p}_\ell(\vec{m})$.

It follows that the relevant factor determining the effect of competition is whether uncommitted followers would benefit or lose from completion of the project/s relative to the status quo; i.e., competition is bad for followers when the externality on uncommitted followers is positive, and beneficial when it is negative. In the case of takeovers, in particular, this says that shareholders would not benefit from competition between raiders. This is also the case in example 4.10 as stated, when legislators would gain if the bill passes without their support. If however we assume that both voters and legislators equally dislike the project, so that $z = w < 0$, competition between lobbyists would increase legislators' welfare. In our exclusive deals example 4.11, there are negative externalities on uncommitted agents, $w_j < 0$, and competition increases followers' welfare.

4.3. **Robustness.** We end this section by discussing two key assumptions in the model: (un)observability of trades and (in)direct competition.[22]

Unobservable Trades. A core assumption in our model is that past deals are observable, so that the state, m or $\vec{m}$, is common knowledge. This assumption captures what we believe is an important dynamic aspect of these problems. The assumption, however, is not neutral for followers' equilibrium payoffs. Common knowledge about the state affects outcomes in both monopoly and competition because the state affects the value

[22]In Appendix 7.3 we discuss an extension of the model in which followers can choose to reject offers permanently, leaving the pool of uncommitted followers, and in Appendix 7.4 we revisit our assumption that leaders offer cash transfers in exchange for a commitment of support, allowing instead promises of transfers contingent on winning (a "partnership" offer instead of a buyout).

of free riding opportunities, and thus agents' bargaining power. Whether unobservability would upend or reinforce the results in the benchmark model depends on whether this increases agents' bargaining power in competition significantly more than what it does in monopoly, or the reverse is true. To explore this question, in Appendix 7.1 we analyze the monopoly and competitive games under non-observability for $n = 3$, following Noe and Wang (2004). We show that still in this case, monopoly is preferred to competition. This is because unobservability of trades doesn't change the underlying fact that the expected value of free riding opportunities (now also over states) is larger under monopoly than under competition.

Direct Competition. In our model we assumed a sequential contracting setup, in which leaders and followers make deals in bilateral meetings. We believe that this is a natural setup for the applications we have in mind, where individual deals are more common than organized markets. A consequence of this assumption is that competition is "indirect" in the sense that it affects trades only through its effect on followers' outside options.[23] In Appendix 7.2 we introduce the possibility of *direct competition*; i.e., w.p. $\pi_{AB} > 0$, both leaders make simultaneous offers to the followers. Direct competition improves the standing of competition vis a vis monopoly, because followers can extract additional rents from the winning leader. This new effect now competes with the free-riding effect which favors monopoly, and can overturn it for some configurations of parameters. But because additional rents are heavily captured by agents at the later stages in the game, followers see a reduced advantage from direct competition in early stages. Because of this, our results are quite robust to the presence of direct competition, even for large π_{AB} when there are no frictions (for δ close to one).

5. Conclusion

In this paper, we consider a class of dynamic collective action problems in which either a single monopolistic principal or two competing principals vie for the support of members of a group. We show that when the principals' policy provides value to the agents, competition reduces agents' welfare. The key to this result is that competition hurts agents because it reduces their free-riding opportunities.

[23]This is how competition among proposers enters in the vast majority of collective bargaining models, including *all* models in the tradition of Baron and Ferejohn (1989) and Chatterjee et al. (1993).

In the main part of the paper we worked with a stripped-down model in which followers' payoffs in terminal nodes are unaffected by whether they supported the winner, the loser, or remained uncommitted. We then generalized this model, allowing followers' payoffs to depend on whether each follower backed a winning or losing alternative. The expanded model shows that the key consideration for our result is whether an uncommitted follower would benefit or lose from completion; i.e., competition is bad for followers when the externality on uncommitted followers is positive, and beneficial when it is negative. Competition reduces agents' welfare when there are positive externalities on uncommitted followers precisely because in this case it reduces agents' free riding opportunities.

Much work remains ahead. In Section 4.3 we discussed non-observability of trades and direct competition. Both suggest promising avenues for further research.

A major direction for future research is a systematic treatment of competitive sequential contracting with heterogeneity among followers. In our model we have assumed agents are ex ante homogeneous. This seems a reasonable first step. First, because agents are ex ante homogeneous, any differential treatment an agent obtains in equilibrium is only due to the state in which he bargains with the leader/s. This allows us to isolate cleanly the core mechanism in the model. Second, having a homogeneous pool of agents leads to an unambiguous comparison of agents' welfare under monopoly and competition. Third, this assumption simplifies the analysis considerably. The difficulty is that heterogeneity increases the size of the state space considerably. In addition to tracking the number of followers that each leader needs to win, the state space must now also keep track of the identity and characteristics of each one of the uncommitted followers. This becomes relevant as leaders may want to exclude some agents from negotiations in order to extract more surplus from other followers, introducing strategic distributional considerations. Potential equilibrium strategies, then, quickly become very complicated as n increases, as trades depend on a very large state space. With this caveat in mind, we believe that a full analysis of competitive sequential contracting with heterogeneity among followers will open interesting questions. We leave this endeavor for future research.

References

Baron, David P. and John A. Ferejohn, "Bargaining in Legislatures," *American Political Science Review*, December 1989, *83*, 1181–1206.

Bergstrom, Theodore, Lawrence Blume, and Hal Varian, "On the private provision of public goods," *Journal of public economics*, 1986, *29* (1), 25–49.

Bloch, Francis, "Sequential formation of coalitions in games with externalities and fixed payoff division," *Games and Economic Behavior*, 1996, *14* (1), 90–123.

Chatterjee, Kalyan, Bhaskar Dutta, Debraj Ray, and Kunal Sengupta, "A noncooperative theory of coalitional bargaining," *The Review of Economic Studies*, 1993, *60* (2), 463–477.

Dal Bo, Ernesto, "Bribing Voters," *American Journal of Political Science*, October 2007, *51*, 789–803.

Dekel, Eddie, Matthew Jackson, and Asher Wolinsky, "Vote Buying: General Elections," *Journal of Political Economy*, April 2008, *116* (2), 351–380.

______, ______, and ______, "Vote Buying: Legislatures and Lobbying," *Quarterly Journal of Political Science*, 2009, *4*, 103–128.

Galasso, Alberto, "Coordination and bargaining power in contracting with externalities," *Journal of Economic Theory*, 2008, *143* (1), 558–570.

Genicot, Garance and Debraj Ray, "Contracts and Externalities: How Things Fall Apart," *Journal of Economic Theory*, 2006, *131*, 71–100.

Ghosh, Suman, Alexander Karaivanov, and Mandar Oak, "A case for bundling public goods contributions," *Journal of Public Economic Theory*, 2007, *9* (3), 425–449.

Gomes, Armando, "Multilateral contracting with externalities," *Econometrica*, 2005, *73* (4), 1329–1350.

______ and Philippe Jehiel, "Dynamic processes of social and economic interactions: on the persistence of inefficiencies," *Journal of Political Economy*, 2005, *113* (3), 626–667.

Groseclose, Tim and James M. Snyder, "Buying Supermajorities," *American Political Science Review*, June 1996, *90*, 303–315.

Grossman, Sanford J and Oliver Hart, "Takeover Bids, the Free-Rider Problems, and the Theory of the Corporation," *Bell Journal of Economics*, 1980, *11* (1), 42–64.

Harrington, Joseph E and Jacek Prokop, "The dynamics of the free-rider problem in takeovers," *Review of Financial Studies*, 1993, *6* (4), 851–882.

Harris, Christopher and John Vickers, "Perfect Equilibrium in a Model of Race," *The Review of Economic Studies*, 1985, *52* (2), 193–209.

______ and ______, "Racing with Uncertainty," *The Review of Economic Studies*, 1987, *54* (1), 1–21.

Holmstrom, Bengt and Barry Nalebuff, "To the Raider Goes the Surplus? A Reexamination of the Free-Rider Problem," *Journal of Economics & Management Strategy*, 1992, *1* (1), 37–62.

Iaryczower, Matias and Santiago Oliveros, "Power Brokers: Middlemen in Legislative Bargaining," *Journal of Economic Theory*, 2016, *162*, 209–236.

Möller, Marc, "The timing of contracting with externalities," *Journal of Economic Theory*, 2007, *133* (1), 484–503.

Myerson, Roger B., " Incentives to Cultivate Favored Minorities Under Alternative Electoral Systems," *American Political Science Review*, December 1993, *87*, 856–869.

Noe, Thomas H and Jun Wang, "Fooling all of the people some of the time: a theory of endogenous sequencing in confidential negotiations," *The Review of Economic Studies*, 2004, *71* (3), 855–881.

Olson, Mancur, *Power and prosperity: Outgrowing communist and capitalist dictatorships: Outgrowing communist and capitalist dictatorships*, Basic books, 2000.

Prat, Andrea and Aldo Rustichini, "Games played through agents," *Econometrica*, 2003, *71* (4), 989–1026.

Rasmusen, Eric and J Mark Ramseyer, "Cheap bribes and the corruption ban: A coordination game among rational legislators," *Public Choice*, 1994, *78* (3-4), 305–327.

Rasmusen, Eric B, J Mark Ramseyer, and John S Wiley Jr, "Naked Exclusion," *The American Economic Review*, 1991, pp. 1137–1145.

Ray, Debraj and Rajiv Vohra, "A theory of endogenous coalition structures," *Games and Economic Behavior*, 1999, *26* (2), 286–336.

——— **and** ———, "Coalitional power and public goods," *Journal of Political Economy*, 2001, *109* (6), 1355–1385.

Rubinstein, Ariel, "Perfect equilibrium in a bargaining model," *Econometrica*, 1982, pp. 97–109.

Segal, Ilya, "Contracting with Externalities," *Quarterly Journal of Economics*, 1999, *114* (2), 337–388.

———, "Coordination and discrimination in contracting with externalities: Divide and conquer?," *Journal of Economic Theory*, 2003, *113* (2), 147–181.

Segal, Ilya R and Michael D Whinston, "Naked exclusion: comment," *American Economic Review*, 2000, pp. 296–309.

6. Online Appendix A: Proofs

Proof of Proposition 3.1. We begin allowing a MPE in mixed strategies. When the leader meets follower i in state m, she makes an offer $p(m)$ with probability $\gamma_m \in [0,1]$. The follower accepts the offer with probability $\alpha_m \in [0,1]$. Note that the follower i meeting the leader in state m accepts only if $\delta w_{out}(m-1) + p(m) \geq \delta w(m)$, and accepts with probability one if this inequality holds strictly. Note that since i accepts offers $p(m) > -\delta[w_{out}(m-1) - w(m)]$ with probability one, then any such proposal cannot be offered in equilibrium, for L could make a lower offer and still get accepted. Thus, whenever L meets a follower i in state m, she offers

$$p(m) = \begin{cases} -\delta[w_{out}(m-1) - w(m)] & \text{if (7) holds} \\ -\infty & \text{otherwise.} \end{cases} \tag{6}$$

L is willing to make the offer in state m if

$$\alpha_m[\delta v(m-1) - p(m)] + (1-\alpha_m)\delta v(m) \geq \delta v(m),$$

which boils down to

$$p(m) \leq \delta[v(m-1) - v(m)],$$

as before. Thus the leader obtains a non-negative payoff from making an offer if and only if

$$s(m) \equiv [v(m-1) - v(m)] + [w_{out}(m-1) - w(m)] \geq 0 \tag{7}$$

Now suppose that in equilibrium (7) holds strictly in state m. Then the follower meeting the leader in state m must accept all such offers; i.e., $\alpha_m = 1$. This is because since the follower accepts any offer higher than $-\delta[w_{out}(m-1) - w(m)]$, if $\alpha_m < 1$ the leader would increase the offer slightly, getting a discrete gain in payoffs. Thus, if in equilibrium the follower rejects the leader's offer with positive probability in state m, (7) must hold with equality in state m; i.e., if $\alpha_m < 1$, then

$$s(m) = [v(m-1) - v(m)] + [w_{out}(m-1) - w(m)] = 0$$

The value of an uncommitted follower in state m is

$$w(m) = \left(\frac{2}{n+2m-1}\right)\delta w(m) + \left(\frac{n+2m-3}{n+2m-1}\right)\delta[\gamma_m\alpha_m w\,(m-1) + (1-\gamma_m\alpha_m)w\,(m)],$$

or equivalently,

$$w(m) = H_m \delta w\,(m-1)\,, \tag{8}$$

where

$$H_m \equiv \left(\frac{(n+2m-3)\gamma_m\alpha_m}{n+2m-1-2\delta-(n+2m-3)\delta(1-\gamma_m\alpha_m)}\right)$$

Thus

$$w(m) = \left[\prod_{k=1}^{m} H_k\right]\delta^m w \tag{9}$$

The value of a committed follower in state m is

$$w_{out}\,(m) = \gamma_m\alpha_m\delta w_{out}\,(m-1) + (1-\gamma_m\alpha_m)\delta w_{out}\,(m)$$

or

$$w_{out}\,(m) = \left(\frac{\gamma_m\alpha_m\delta}{1-\delta(1-\gamma_m\alpha_m)}\right)w_{out}\,(m-1)$$

so that

$$w_{out}\left(m\right)=\left[\prod_{k=1}^{m}\left(\frac{\gamma_k\alpha_k}{1-\delta(1-\gamma_k\alpha_k)}\right)\right]\delta^m w. \tag{10}$$

The value for the leader in state m is

$$v(m)=\gamma_m\alpha_m(\delta v(m-1)-p(m))+(1-\gamma_m\alpha_m)\delta v(m)$$

or

$$v(m)=\left(\frac{\gamma_m\alpha_m\delta}{1-\delta(1-\gamma_m\alpha_m)}\right)(v(m-1)+w_{out}(m-1)-w(m)), \tag{11}$$

Now suppose that in equilibrium L makes a relevant offer in every $m>1$. We will solve for the equilibrium values and then come back and verify that (7) holds for all m to check that this is an equilibrium. First, note that since L makes a relevant offer in every meeting, (9) boils down to

$$w(m)=\left[\prod_{k=1}^{m}\left(\frac{n+2k-3}{n+2k-1-2\delta}\right)\right]\delta^m w=\left[\prod_{k=1}^{m}r(k)\right]\delta^m w \tag{12}$$

and (10) boils down to

$$w_{out}\left(m\right)=\delta^m w. \tag{13}$$

Substituting (12) and (13) in (11), we have

$$v\left(m\right)=\delta v\left(m-1\right)+\left(1-\delta\prod_{k=1}^{m}r(k)\right)\delta^m w$$

Recursively we have that

$$v\left(m\right)=\delta^m v+\left[\sum_{l=1}^{m}\left(1-\delta\prod_{k=1}^{l}r(k)\right)\right]\delta^m w \tag{14}$$

Then note that

$$v(m-1)-v(m)=\delta^{m-1}(1-\delta)v+\delta^{m-1}w\left\{(1-\delta)\sum_{l=1}^{m-1}\left(1-\delta\prod_{k=1}^{l}r(k)\right)-\delta\left(1-\delta\prod_{k=1}^{m}r(k)\right)\right\}$$

and

$$w(m)-w_{out}(m-1)=-\left[1-\delta\prod_{k=1}^{m}r(k)\right]\delta^{m-1}w$$

so substituting, (7) is

$$s^*(m)=(1-\delta)\delta^{m-1}\left[v+w\sum_{l=1}^{m}\left(1-\delta\prod_{k=1}^{l}r(k)\right)\right]\geq 0$$

which is satisfied if and only if

$$v+w\sum_{l=1}^{m}\left(1-\delta\prod_{k=1}^{l}r(k)\right)\geq 0$$

Because this always holds for $v > 0$ and $w > 0$, it follows that this is an equilibrium.

Next we show that this is the unique equilibrium with an induction argument. First note from (9) and (11) that for all $m \geq 1$, $v(m)$ and $w(m)$ are maximized when $\gamma_m = \alpha_m = 1$. Then $s^*(1) \geq 0$ implies $s(1) = [v - v(1)] + [w - w(1)] > 0$ whenever $\gamma_1\alpha_1 < 1$. It follows that in state $m = 1$ the leader makes a proposal with probability one; i.e., $\gamma_1 = 1$. But then $\alpha_1 = 1$ as well. For suppose $\alpha_1 \in (0, 1)$. Then $s(1) > 0$ and the leader would gain by increasing the offer slightly, getting it accepted with probability one. Now suppose that in equilibrium $\gamma_t = \alpha_t = 1$ for all $t < m$. Consider the surplus in state m. Note that $v(m-1)$ and $w_{out}(m-1)$ are exactly as in the equilibrium characterized above. Since $v(m)$ and $w(m)$ are maximized when $\gamma_m = \alpha_m = 1$, then $s^*(m) \geq 0$ implies $s(m) > 0$ whenever $\gamma_m\alpha_m < 1$. Thus $\gamma_m = 1$. As before, then also $\alpha_1 = 1$, for otherwise $s(m) > 0$ and the leader would gain by increasing the offer slightly, getting it accepted with probability one. ■

Proof of Proposition 4.1. Part (i) follows from standard arguments (click here for details). Next consider part (ii). Let $\beta(\vec{m})$ denote the probability that any given uncommitted follower meets with one of the leaders. Note if ℓ has to secure the support of m_ℓ more followers there are $(n+1) - m_A - m_B$ committed followers, and $m_A + m_B - 1$ uncommitted followers. Then $\beta(\vec{m}) = 1/(m_A + m_B - 1)$. As in the examples, eq. (1) implies that the expected payoff of a follower after meeting one of the leaders is $\delta W(\vec{m})$ independently of whether he accepts the proposal or not. This is a crucial property, for it allows us to decouple the system of partial difference equations for $W(\vec{m})$ and $V_\ell(\vec{m})$, $\ell = A, B$. Then

$$W(\vec{m}) = \left(\frac{1}{m_A + m_B - 1}\right) \delta W(\vec{m}) + \left(\frac{m_A + m_B - 2}{m_A + m_B - 1}\right) \delta \sum_\ell \pi_\ell W(\vec{m}^\ell), \tag{15}$$

so that letting $C(k) \equiv \frac{k-2}{k-(1+\delta)}$, we have

$$W(\vec{m}) = C(m_A + m_B)\delta \sum_\ell \pi_\ell W(\vec{m}^\ell). \tag{16}$$

Equation (16) is a partial difference equation with end points $W(m_A, 0) = w_B$ for $m_A > 0$ and $W(0, m_B) = w_A$ for $m_B > 0$, and $W(1, 1) = 0$, which we can solve to obtain (3).

Having obtained the expression for $W(\vec{m})$ in terms of the fundamentals, we can write down equilibrium transfers. Note that once a follower is committed, all strategic considerations are brushed aside, as a committed follower just needs to wait for a leader to form a majority. Thus

$$W_{out}(\vec{m}) = \sum_{j=A,B} (\delta\pi_j)^{m_j} \times \left(\sum_{l=0}^{m_{-j}-1} \binom{m_j - 1 + l}{l} \times (\delta\pi_{-j})^l\right) \times w_j \tag{17}$$

From equation (1), expressions (3) and (17) pin down equilibrium transfers $p_\ell(\vec{m})$ in terms of the fundamentals. This in turn allows us to solve for the value of the leaders, which is given, in recursive form, by

$$V_\ell(\vec{m}) = \pi_\ell \left(\delta V_\ell\left(\vec{m}^\ell\right) - p_\ell(\vec{m})\right) + (1 - \pi_\ell)\delta V_\ell\left(\vec{m}^{-\ell}\right) \tag{18}$$

Once we write transfers in terms of the primitives, (18) becomes a stand alone difference equation, which we can solve. This allows us to prove the next result.

We will show that for any $j = A, B$ there is a $v^* \in \mathbb{R}_+$ such that if $\overline{v}_j \geq v^*$, when all players play the proposed equilibrium strategies, $S_j(\vec{m}) \geq 0$ for all $\vec{m}$.

Consider the surplus expression (2). Note that (3) and (17) imply that $W_{out}(\vec{m}^j) - W(\vec{m})$ does not depend on $(\overline{v}_A, \underline{v}_A, \overline{v}_B, \underline{v}_B)$. It follows that $\overline{v}_{-j}$ and $\underline{v}_{-j}$ do not affect $S_j(\vec{m})$, and $\overline{v}_j$ and $\underline{v}_j$ enter $S_j(\vec{m})$ only through the term $V_j(\vec{m}^j) - V_j(\vec{m})$. Now, note that having expressed $p_j(\vec{m})$ in terms of the primitives of the model, we can solve (18) as a stand alone partial difference equation, to obtain

$$V_j(\vec{m}) = (\delta\pi_j)^{m_j}\left[\sum_{l=0}^{m_{-j}-1}\binom{m_j-1+l}{l}(\delta\pi_{-j})^l\right]\overline{v}_j + (\delta\pi_{-j})^{m_{-j}}\left[\sum_{l=0}^{m_j-1}\binom{m_{-j}-1+l}{l}(\delta\pi_j)^l\right]\underline{v}_j - H(\vec{m}). \tag{19}$$

where $H(\vec{m})$ is a function of prices $p_j(r,s)$ for $r \leq m_j, s \leq m_{-j}$, which are constant in $(\overline{v}_j, \underline{v}_j, \overline{v}_{-j}, \underline{v}_{-j})$ by (3) and (17). Thus $V_j(\vec{m}^j) - V_j(\vec{m})$ is given by

$$\left\{(\delta\pi_j)^{m_j-1}\left[\sum_{l=0}^{m_{-j}-1}\binom{m_j-2+l}{l}(\delta\pi_{-j})^l\right] - (\delta\pi_j)^{m_j}\left[\sum_{l=0}^{m_{-j}-1}\binom{m_j-1+l}{l}(\delta\pi_{-j})^l\right]\right\}\overline{v}_j$$

$$-(\delta\pi_{-j})^{m_{-j}}\binom{m_A+m_B-2}{m_j-1}(\delta\pi_j)^{m_j-1}\underline{v}_j + H(\vec{m}) - H(\vec{m}^j).$$

We will show that this expression can be made arbitrarily large by increasing $\overline{v}_j$ or reducing $\underline{v}_j$. From the second line it follows that all else equal, there is a $\underline{v}^*$ such that if $\underline{v}_j < \underline{v}^*$, then $S_j(\vec{m}) > 0$. Next, after some algebra, the bracket in the first line can be written as

$$(\delta\pi_j)^{m_j-1}\left[(1-\delta)\sum_{l=0}^{m_{-j}-1}\binom{m_j-1+l}{l}(\delta\pi_{-j})^l + \binom{m_A+m_B-2}{m_j-1}(\delta\pi_{-j})^{m_{-j}}\right] > 0.$$

Thus, all else equal, there is a $\overline{v}^*$ such that if $\overline{v}_j > \overline{v}^*$, then $S_j(\vec{m}) > 0$. ■

Proof of Theorem 4.3. Let $\gamma_j(\vec{m})$ be the probability that leader $j = A, B$ makes an offer in state $\vec{m}$, $\alpha_j(\vec{m})$ be the probability that an uncommitted follower accepts an offer from leader $j = A, B$ in state $\vec{m}$, and $\mu_j(\vec{m}) \equiv \gamma_j(\vec{m})\alpha_j(\vec{m})$. Then

$$W(\vec{m}) = \left(\frac{1}{m_A+m_B-1}\right)\delta W(\vec{m}) + \left(\frac{m_A+m_B-2}{m_A+m_B-1}\right)\sum_{j=A,B}\pi_j\begin{pmatrix}\mu_j(\vec{m})\,\delta W(\vec{m}^j)\\ +(1-\mu_j)\,\delta W(\vec{m})\end{pmatrix}. \tag{20}$$

For $j = A, B$, define

$$\xi_j(\vec{m}) \equiv \frac{\delta \pi_j \mu_j(\vec{m})}{\left(\frac{m_A+m_B-1}{m_A+m_B-2}\right)(1-\delta) + \delta \sum_{j=A,B} \pi_j \mu_j(\vec{m})}$$

whenever $\vec{m} \neq (1,1)$, and $\xi_j(1,1) \equiv 0$. Then we can write (20) as

$$W(\vec{m}) = \sum_{j=A,B} \xi_j(\vec{m}) W(\vec{m}^j) \tag{21}$$

for all $\vec{m}$ and $j = A, B$. Note in particular that the recursion (21) implies that if $w_A, w_B > 0$ (as we are assuming here), then $W(\vec{m}) \geq 0$ for all $\vec{m}$.

We need to show that $W(q,q) < \max\{w_A(q), w_B(q)\}$. The proof follows from three lemmas. Lemma A.2 establishes the result for $q = 1$ and shows an additional result for all boundary states which is used in Lemma A.3. The proof for interior states is by induction. Lemma A.3 establishes the base case, and Lemma A.4 the induction step. Iterative application of the induction step covers the entire state space and establishes the result. We begin with Lemma A.1, which establishes an intermediate result that is used in the proof of Lemmas A.2 and A.4. ■

Lemma A.1 (Bound)**.** *In any MPE of the game* $\Gamma(\vec{m})$*,*

$$W(\vec{m}) \leq \max_{j \in \{A,B\}} \left\{\delta r(m_j) W(\vec{m}^j)\right\}$$

Proof of Lemma A.1. Note that for all $m_A \geq 2, m_B \geq 2$ we have

$$W(\vec{m}) = \xi_A(\vec{m}) W(\vec{m}^A) + \xi_B(\vec{m}) W(\vec{m}^B)$$

Thus we need to show that

$$\sum_{j=A,B} \xi_j(\vec{m}) W(\vec{m}^j) \leq \delta \max\left\{r(m_A) W(\vec{m}^A), r(m_B) W(\vec{m}^B)\right\}$$

Without loss of generality assume that $W(\vec{m}^A) \geq W(\vec{m}^B)$ so it is sufficient if

$$\left[\sum_{j=A,B} \xi_j(\vec{m})\right] W(\vec{m}^A) \leq \delta \max\left\{r(m_A) W(\vec{m}^A), r(m_B) W(\vec{m}^B)\right\}$$

Note that since $r(m) = \frac{n+2m-3}{n+2m-(1+2\delta)}$, then

$$\begin{aligned}\left[\sum_{j=A,B} \xi_j(\vec{m})\right] &= \frac{\delta\left[\pi_A \mu_A(\vec{m}) + \pi_B \mu_B(\vec{m})\right]}{\frac{m_A+m_B-1}{m_A+m_B-2}(1-\delta) + \delta\left[\pi_A \mu_A(\vec{m}) + \pi_B \mu_B(\vec{m})\right]} \\ &\leq \delta \min\left\{r(m_A), r(m_B)\right\}.\end{aligned} \tag{22}$$

Then it is sufficient if

$$\min\left\{r(m_A), r(m_B)\right\} W(\vec{m}^A) \leq \max\left\{r(m_A) W(\vec{m}^A), r(m_B) W(\vec{m}^B)\right\}$$

which is true when either $r(m_A)W(\vec{m}^A) \geq r(m_B)W(\vec{m}^B)$ or the opposite holds. ■

Lemma A.2 (Boundaries)**.**

$$W(m_A,1) < w_B(1) \text{ for all } m_A \geq 1 \quad \text{and} \quad W(1,m_B) < w_A(1) \text{ for all } m_B \geq 1$$

Proof of Lemma A.2. The result for state $\vec{m} = (1,1)$ follows immediately from the fact that $W(1,1) = 0$. Now consider the remaining boundary states (states adjacent to terminal states). Solving the recursion (21) for the boundaries, we obtain

$$W(m_A,1) = \left(\sum_{l=1}^{m_A-1} \xi_B(m_A-l,1)\prod_{k=0}^{l-1}\xi_A(m_A-k,1) + \xi_B(m_A,1)\right)w_B \tag{23}$$

$$W(1,m_B) = \left(\sum_{l=1}^{m_B-2} \xi_A(1,m_B-l)\prod_{k=0}^{l-1}\xi_B(1,m_B-k) + \xi_A(1,m_B)\right)w_A \tag{24}$$

for all $m_A, m_B \geq 1$.

Consider $\vec{m} = (2,1)$. Note that since

$$W(2,1) = \frac{\delta\pi_B\mu_B(2,1)}{2(1-\delta)+\delta(\pi_A\mu_A(2,1)+\pi_B\mu_B(2,1))}w_B$$

for $n \geq 3$,

$$W(2,1) < \frac{\delta}{2-\delta}w_B < \frac{(n-1)\delta}{n+1-2\delta}w_B = r(1)\delta w_B = w_B(1)$$

By the same argument, $W(1,2) < w_A(1)$. Next, consider $W(m_A,1)$ for $m_A \geq 3$. We have

$$\begin{aligned} W(m_A,1) &= \left(\sum_{l=0}^{m_A-3}\xi_B(m_A-(1+l),1)\prod_{k=0}^{l}\xi_A(m_A-k,1)+\xi_B(m_A,1)\right)w_B \\ &= \left(\begin{array}{c}\sum_{l=0}^{m_A-3}\left[\xi_B(m_A-(1+l),1)+\xi_A(m_A-(1+l),1)-1\right]\prod_{k=0}^{l}\xi_A(m_A-k,1) \\ -\prod_{k=0}^{m_A-2}\xi_A(m_A-k,1)+(\xi_B(m_A,1)+\xi_A(m_A,1))\end{array}\right)w_B, \end{aligned}$$

and since $(\xi_B(m_A-(1+l),1)+\xi_A(m_A-(1+l),1)) \leq 1$, it follows that

$$W(m_A,1) \leq (\xi_B(m_A,1)+\xi_A(m_A,1))w_B < \delta r(1)w_B = w_B(1)$$

Analogously, we have that $W(1,m_B) < w_A(1)$. ■

Lemma A.3 (Base Case)**.**

$$W(m_A,2) < \max\{w_A(m_A), w_B(2)\} \quad \text{for all } m_A \geq 2$$

and

$$W(2,m_B) < \max\{w_A(2), w_B(m_B)\} \quad \text{for all } m_B \geq 2$$

Proof of Lemma A.3. First, note that

$$\begin{aligned} W(2,2) &\leq \xi(4)\max\{w_B(1), w_A(1)\} \\ &< \delta\left(\frac{2}{3-\delta}\right)\max\{w_B(1), w_A(1)\} \\ &< \delta r(2)\max\{w_B(1), w_A(1)\} = \max\{w_B(2), w_A(2)\} \end{aligned}$$

Next consider $W(m_A, 2)$. By successive application of Lemma A.1

$$\begin{aligned} W(m_A,2) &\leq \max\{\delta r(m_A) W(m_A-1,2), \delta W(m_A,1) r(2)\} \\ &\leq \max\left\{\delta^2 \prod_{j=0}^{1} r(m_A-j) W(m_A-2,2), \delta^2 \prod_{j=0}^{0} r(m_A-j) W(m_A-1,1) r(2), \delta W(m_A,1) r(2)\right\} \\ &.... \\ &\leq \max\left\{\delta^{m_A} \prod_{j=0}^{m_A-2} r(m_A-j) W(1,2), \max_{k\leq m_A-2}\left\{\delta^{k+1}\prod_{j=0}^{k-1} r(m_A-j) W(m_A-k,1) r(2)\right\}, \delta W(m_A,1) r(2)\right\} \end{aligned}$$

Now, we have shown in Lemma A.2 that $W(m_A, 1) < w_B(1)$ for all $m_A \geq 1$, and $W(1, m_B) < w_A(1)$ for all $m_B \geq 1$. Using these results in the RHS of the expression above, we get

$$W(m_A,2) < \max\left\{\delta^{m_A} \prod_{j=0}^{m_A-2} r(m_A-j) w_A(1), \ \max_{k\leq m_A-2}\left\{ \ \delta^{k+1}\prod_{j=0}^{k-1} r(m_A-j) w_B(1) \ \right\} r(2) \ , \delta r(2) w_B(1)\right\}$$

Using (12) we get that

$$\max_{k\leq m_A-2}\left\{\delta^{k+1}\prod_{j=0}^{k-1} r(m_A-j) w_B(1)\right\} = \delta^2 r(m_A) w_B(1),$$

so

$$W(m_A,2) < \max\left\{\delta^{m_A}\prod_{j=0}^{m_A-2} r(m_A-j) w_A(1), \delta r(2) w_B(1)\right\}.$$

Therefore, using equation (12) and Lemma A.2 one more time, we have

$$W(m_A,2) < \max\{w_A(m_A), w_B(2)\}.$$

By the same logic, $W(2, m_B) < \max\{w_B(m_B), w_A(2)\}$. ∎

Lemma A.4 (Induction Step). *Consider any state $\vec{m} \geq (3, 3)$. If*

(25) $$W\left(\vec{m}^B\right) \leq \max\{w_A(m_A), w_B(m_B-1)\}$$

and

(26) $$W\left(\vec{m}^A\right) \leq \max\{w_A(m_A-1), w_B(m_B)\}$$

then

(27) $$W(\vec{m}) \leq \max\{w_A(m_A), w_B(m_B)\}$$

Proof of Lemma A.4. By Lemma A.1,

(28) $$W(\vec{m}) \leq \max\left\{\delta r(m_A) W\left(\vec{m}^A\right), \delta r(m_B) W\left(\vec{m}^B\right)\right\}$$

Using (25) and (26), and then noting that $w_j(m_j) = \delta r(m_j) w_j(m_j-1)$ for $j = A, B$, and substituting, (28) becomes

$$W(\vec{m}) \leq \max \left\{ \begin{array}{c} \max\{w_A(m_A), \delta r(m_A) w_B(m_B)\}, \\ \\ \max\{\delta r(m_B) w_A(m_A), w_B(m_B)\} \end{array} \right\} \leq \max\{w_A(m_A), w_B(m_B)\}$$

■

Proof of Theorem 4.5. The first statement follows as a corollary of Theorem 4.3. Now consider the second part. From expression (3), we have

$$W(q,q) = \sum_{l=0}^{q-2} \left(\prod_{k=0}^{q-1+l} C(2q-k) \right) \times \binom{q-1+l}{l} \times \left[(\delta\pi_A)^q (\delta\pi_B)^l w_A + (\delta\pi_B)^q (\delta\pi_A)^l w_B \right] \tag{29}$$

On the other hand, with a single alternative, $w(q) = \left(\prod_{m=1}^{q} r(m)\right) \delta^q w$. Now, since $r(k) = \frac{n+2k-3}{n+2k-(1+2\delta)}$ by definition and $n = 2q-1$, we have $r(k) = C(q+k)$. Thus

$$w(q) = \left(\prod_{k=1}^{q} C(q+k) \right) \delta^q w = \left(\prod_{k=0}^{q-1} C(2q-k) \right) \delta^q w \tag{30}$$

Suppose without loss of generality that $w_A > w_B$. We want to show that for sufficiently large q the equilibrium payoff of an uncommitted follower in the game with a single alternative yielding value w_B is larger than his (competitive) equilibrium payoff in the game with two alternatives yielding value w_A and w_B. Suppose not. Then making $w = w_B$ in (30), and dividing (29) by (30),

$$U(q) \equiv \sum_{l=0}^{q-2} \left(\prod_{k=q}^{q-1+l} C(2q-k) \right) \times \binom{q-1+l}{l} \times \left[(\pi_A)^q (\delta\pi_B)^l + (\pi_B)^q (\delta\pi_A)^l \left(\frac{w_B}{w_A} \right) \right] \geq \frac{w_B}{w_A}$$

Now, since $\delta \leq 1$, $\prod_{k=q}^{q-1+l} C(2q-k) \leq 1$, and $w_B/w_A < 1$, for any integer q, we have

$$U(q) < \sum_{j=A,B} (\pi_j)^q \sum_{l=0}^{q-2} \frac{\Gamma(q+l)}{\Gamma(q)} \frac{(\delta\pi_j)^l}{l!} \equiv \overline{U}(q)$$

where for any integer k, we define $\Gamma(k) \equiv (k-1)!$. Now define the function

$$F(a,b,c,z) \equiv \sum_{l=0}^{\infty} \frac{\binom{a+l}{l}\binom{b+l}{l}}{\binom{c+l}{l}} z^l,$$

and note that we can write

$$\begin{aligned} \overline{U}(q) &= \sum_{j=A,B} (\pi_j)^q \left(F(q,1,1,\delta\pi_{-j}) - \sum_{l=q-1}^{\infty} \frac{\Gamma(q+l)}{\Gamma(q)} \frac{(\delta\pi_{-j})^l}{l!} \right) \\ &= \sum_{j=A,B} \left\{ \left(\frac{\pi_j}{1-\delta\pi_{-j}} \right)^q - (\pi_j)^q \sum_{l=q-1}^{\infty} \frac{\Gamma(q+l)}{\Gamma(q)} \frac{(\delta\pi_{-j})^l}{l!} \right\}. \end{aligned} \tag{31}$$

where the equality follows from the fact that $F(a,b,b,z) = (1-z)^{-a}$ (see Property 15.1.8 for hypergeometric functions in Abramowitz and Stegun (2012; p.556)). Noting that

$$\left(\frac{\pi_j}{1-\delta\pi_{-j}}\right) = \left(\frac{1-\pi_{-j}}{1-\delta\pi_{-j}}\right) < 1$$

as long as $\delta < 1$, it follows that for any $\varepsilon > 0$ there is a Q such that if $q > Q$, then $\overline{U}(q) < \varepsilon$. Thus, for any π_B/π_A, there is a Q such that $U(q) < \pi_B/\pi_A$ whenever $q > Q$. ■

Proof of Theorem 4.6. Let $\mathcal{L}$ be the set of leaders with $|\mathcal{L}| \geq 2$. Let $\gamma_j(\vec{m})$ be the probability that leader $j \in \mathcal{L}$ makes an offer in state $\vec{m}$, $\alpha_j(\vec{m})$ be the probability that an uncommitted follower accepts an offer from leader $j \in \mathcal{L}$ in state $\vec{m}$, and $\mu_j(\vec{m}) \equiv \gamma_j(\vec{m})\alpha_j(\vec{m})$. Then

$$W(\vec{m}) = \left(\frac{1}{n-\sum_{j\in\mathcal{L}}\left(\frac{n+1}{2}-m_j\right)}\right)\delta W(\vec{m}) + \left(1-\frac{1}{n-\sum_{j\in\mathcal{L}}\left(\frac{n+1}{2}-m_j\right)}\right)\sum_{j\in\mathcal{L}}\pi_j\left(\begin{array}{c}\mu_j(\vec{m})\,\delta W\left(\vec{m}^j\right)\\ +\left(1-\mu_j\right)\delta W(\vec{m})\end{array}\right) \tag{32}$$

Let $M^+ \equiv \{\vec{m} : n-\sum_{j\in\mathcal{L}}\left(\frac{n+1}{2}-m_j\right) > \min_{j\in\mathcal{L}} m_j\}$ denote the set of states in which not all uncommitted followers are critical. For $j \in \mathcal{L}$ and all $\vec{m} \in M^+$ define

$$\xi_j(\vec{m}) \equiv \frac{\delta\pi_j\mu_j(\vec{m})}{\frac{n-\sum_{j\in\mathcal{L}}\left(\frac{n+1}{2}-m_j\right)}{n-1-\sum_{j\in\mathcal{L}}\left(\frac{n+1}{2}-m_j\right)}(1-\delta)+\delta\sum_{j\in\mathcal{L}}\pi_j\mu_j(\vec{m})}$$

For all $j \in \mathcal{L}$ and $\vec{m} \in M^+$ we can write (32) as

$$W(\vec{m}) = \sum_{j\in\mathcal{L}}\xi_j(\vec{m})\,W\left(\vec{m}^j\right) \tag{33}$$

Note in particular that the recursion (33) implies that if $w_j > 0$ for all $j \in \mathcal{L}$ (as we are assuming here), then $W(\vec{m}) \geq 0$ for all $\vec{m} \in M^+$.

We need to show that $W(\vec{q}) < \max_{j\in\mathcal{L}}\{w_j(q)\}$. The proof follows from three lemmas. Lemma A.6 establishes the result for critical states $\vec{m}$, in which at least one leader is exactly one step away from winning, and shows an additional result for all boundary states which is used in Lemma A.7. The proof for interior states is by induction. Lemma A.7 establishes the base case, and Lemma A.8 the induction step. Iterative application of the induction step covers the entire state space and establishes the result. We begin with Lemma A.5, which establishes an intermediate result that is used in the proof of Lemmas A.6 and A.8. ■

Lemma A.5 (Bound)**.** *In any MPE of the game* $\Gamma(\vec{m})$,

$$W(\vec{m}) \leq \max_{j\in\mathcal{L}}\left\{\delta r(m_j)\,W\left(\vec{m}^j\right)\right\} \tag{34}$$

Proof of Lemma A.5. Note that

$$\sum_{j\in\mathcal{L}}\xi_j(\vec{m})\leq\frac{\delta}{\frac{n-\sum_{j\in\mathcal{L}}\left(\frac{n+1}{2}-m_j\right)}{n-1-\sum_{j\in\mathcal{L}}\left(\frac{n+1}{2}-m_j\right)}(1-\delta)+\delta}$$

and since

$$\frac{\delta}{\frac{n-\sum_{j\in\mathcal{L}}\left(\frac{n+1}{2}-m_j\right)}{n-1-\sum_{j\in\mathcal{L}}\left(\frac{n+1}{2}-m_j\right)}(1-\delta)+\delta}\leq\delta\frac{n+2m_k-3}{n+2m_k-(1+2\delta)}=\delta r(m_k)$$

we have that for all $k\in\mathcal{L}$

$$\sum_{j\in\mathcal{L}}\xi_j(\vec{m})\leq\delta r(m_k) \tag{35}$$

Therefore for all $k\in\mathcal{L}$ we have

$$\begin{aligned}\sum_{j\in\mathcal{L}}\xi_j(\vec{m})\max_{j\in\mathcal{L}}\left\{W\left(\vec{m}^j\right)\right\} &\leq \delta r(m_k)\max_{j\in\mathcal{L}}\left\{W\left(\vec{m}^j\right)\right\}\\ \sum_{j\in\mathcal{L}}\xi_j(\vec{m})W\left(\vec{m}^j\right) &\leq \delta\max_{j\in\mathcal{L}}\left\{r(m_j)W\left(\vec{m}^j\right)\right\}\end{aligned}$$

■

Lemma A.6 (Boundaries)**.** *Let $M^*(j)=\{\vec{m}:m_j=1\leq m_k$ for $k\neq j$ and $k\in\mathcal{L}\}$ denote the set of (critical) states in which leader $j\in\mathcal{L}$ is one step away from winning. Let $\vec{m}^*(j)\in M^*(j)$ be a generic element of this set and define $|\vec{m}^*(j)|_h$ as the number of followers that leader $h=A,B,\ldots$ needs to win. Then for all $\vec{m}^*(j)\in M^*(j)$ we have*

$$W(\vec{m}^*(j))\leq\max_{h\in\mathcal{L}}\left\{w_h\left(|\vec{m}^*(j)|_h\right)\right\} \tag{36}$$

and the inequality is strict if $W(\vec{m}^*(j))>0$ is positive.

Proof of Lemma A.6. Without loss of generality, let's focus on $\vec{m}^*(A)\in M^*(A)$. Note that if there is another leader k such that $|\vec{m}^*(j)|_k=1$, we have two leaders that need only one follower to win. Therefore, we must have that there is only one remaining uncommitted follower and $W(\vec{m}^*(A))=0$, which trivially verifies (36).

Let's focus then on the set of states $\widehat{M}^*(A)=\{\vec{m}^*(A)\in M^*(A):m_k\geq 2$ for $k\in\{\mathcal{L}\setminus A\}\}$ in which A is the only leader that needs one supporter while the rest needs at least 2. Note that for any $\vec{m}^*(A)\in\widehat{M}^*(A)$, (33) is equivalent to

$$W(\vec{m}^*(A))=\xi_A(\vec{m}^*(A))w_A+\sum_{j\in\{\mathcal{L}|\mathcal{A}\}}\xi_j(\vec{m}^*(A))W\left(\vec{m}^*(A)^j\right)$$

If $w_A\geq W\left(\vec{m}^*(A)^k\right)$ for all $j\in\{\mathcal{L}\setminus A\}$ we are done since

$$W(\vec{m}^*(A))\leq\sum_{j\in\mathcal{L}}\xi_j(\vec{m}^*(A))w_A<\delta r(1)w_A=w_A(1) \tag{37}$$

Then assume that $w_A<W\left(\vec{m}^*(A)^k\right)$ for some $k\in\{\mathcal{L}\setminus A\}$ so we have

$$W(\vec{m}^*(A))\leq\sum_{j\in\mathcal{L}}\xi_j(\vec{m}^*(A))W\left(\vec{m}^*(A)^k\right)$$

Note that there is no h such that $|\vec{m}^*(A)^h|_h = 1$ since this implies that $|\vec{m}^*(A)|_h = 2$, and therefore any trade leads to only one remaining uncommitted follower. This implies $W(\vec{m}^*(A)^k) = 0$, which contradicts the assumption that $w_A < W(\vec{m}^*(A)^k)$. Therefore, we must have that $\vec{m}^*(A)^k \in \widehat{M}^*(A)$.

Let's focus then on the state $\vec{m}^*(A)^k$. Using (33) we have

$$W\left(\vec{m}^*(A)^k\right) = \xi_A\left(\vec{m}^*(A)^k\right) w_A + \sum_{j\in\{\mathcal{L}\setminus\{A,k\}\}} \xi_j\left(\vec{m}^*(A)^k\right) W\left([\vec{m}^*(A)^k]^j\right)$$

Again we have that if $w_A \geq W\left([\vec{m}^*(A)^k]^j\right)$ we are done, since

$$W\left(\vec{m}^*(A)^k\right) \leq \sum_{j\in\mathcal{L}} \xi_j\left(\vec{m}^*(A)^k\right) w_A < \delta r(1)\, w_A = w_A(1)$$

so we assume again that $w_A < W\left([\vec{m}(A)^k]^{k'}\right)$ for some $k' \in \{\mathcal{L}\setminus A\}$, which implies that

$$W\left(\vec{m}^*(A)^k\right) \leq \sum_{j\in\mathcal{L}} \xi_j\left(\vec{m}^*(A)^k\right) W\left([\vec{m}^*(A)^k]^{k'}\right)$$

and therefore

$$W(\vec{m}^*(A)) \leq \left\{\sum_{j\in\mathcal{L}} \xi_j(\vec{m}^*(A))\right\} \times \left\{\sum_{j\in\mathcal{L}} \xi_j\left(\vec{m}^*(A)^k\right)\right\} \times W\left([\vec{m}^*(A)^k]^{k'}\right) \tag{38}$$

Note that we can proceed in the same fashion until we reach a state in which another leader needs only one follower or there are only two remaining followers. In the first case we have that the value function is 0 because there is only one remaining uncommitted follower, so we focus in the second case. Call this state $\vec{m}_1^*(A)$ (there are many paths in which this state could be reached but only one path that maximizes it step by step) and note that

$$W(\vec{m}^*(A)) \leq \left\{\sum_{j\in\mathcal{L}} \xi_j(\vec{m}^*(A))\right\} \times \left\{\sum_{j\in\mathcal{L}} \xi_j\left(\vec{m}^*(A)^k\right)\right\} \times \left\{\sum_{j\in\mathcal{L}} \xi_j\left([\vec{m}^*(A)^k]^{k'}\right)\right\} \times ... \times W(\vec{m}_1(A))$$

Note also that the fact that there are only two remaining followers implies that $W\left(\vec{m}_1(A)^j\right) = 0$ so (33)

$$\begin{aligned} W(\vec{m}_1^*(A)) &= \xi_A(\vec{m}_1^*(A))\, w_A + \sum_{j\in\{\mathcal{L}|\mathcal{A}\}} \xi_j(\vec{m}_1^*(A))\, W\left(\vec{m}_1^*(A)^j\right) \\ &= \xi_A(\vec{m}_1^*(A))\, w_A \end{aligned}$$

and

$$W(\vec{m}_1^*(A)) = \xi_A(\vec{m}_1^*(A))\, w_A < \delta r(1)\, w_A = w_A(1)$$

Therefore,

$$\begin{aligned} W(\vec{m}^*(A)) &< \left\{\sum_{j\in\mathcal{L}} \xi_j(\vec{m}^*(A))\right\} \times \left\{\sum_{j\in\mathcal{L}} \xi_j\left(\vec{m}^*(A)^k\right)\right\} \times \left\{\sum_{j\in\mathcal{L}} \xi_j\left([\vec{m}^*(A)^k]^{k'}\right)\right\} \times ... \times w_A(1) \\ &< w_A(1) \end{aligned}$$

Recall that $\left|\vec{m}^*(A)\right|_A = 1$ and it is trivial to see that

$$w_A\left(\left|\vec{m}^*(A)\right|_A\right) \le \max_{j\in\mathcal{L}}\left\{w_j\left(\left|\vec{m}^*(A)\right|_j\right)\right\}$$

which gives (36). ■

Lemma A.7 (Base Case)**.** *Let*

$$M^{**}(j) = \{\vec{m} : m_j = 2 \le m_k \text{ for } k \in \{\mathcal{L}\,|j\}\}$$

be the set of states in which all leaders need at least 2 *supporters to win and leader* $j \in \mathcal{L}$ *needs exactly* 2 *supporters, then*

$$W\left(\vec{m}^{**}(j)\right) < \max_{h\in\mathcal{L}}\left\{w_h\left(\left|\vec{m}^{**}(j)\right|_h\right)\right\} \tag{39}$$

Proof of Lemma A.7. Without loss of generaility assume that $j = A$ and let $L\left(\vec{m}^{**}(A)\right) = \left\{h \in \mathcal{L} : \left|\vec{m}^{**}(A)\right|_h = 2\right\}$ be the set of leaders such that if trading they are one supporter away from winning; note that $A \in L\left(\vec{m}^{**}(A)\right)$. Note that

$$W\left(\vec{m}^{**}(A)\right) = \sum_{k\in L(\vec{m}^{**}(A))}\xi_k\left(\vec{m}^{**}(A)\right)W\left(\vec{m}^{**}(A)^k\right) + \sum_{k\notin L(\vec{m}^{**}(A))}\xi_k\left(\vec{m}^{**}(A)\right)W\left(\vec{m}^{**}(A)^k\right)$$

If the state $\vec{m}^{**}(A)$ involves only one remaining uncommitted follower we have that $W\left(\vec{m}^{**}(A)\right) = 0$, so we are done. Assume then that there are more than one uncommitted follower. If there is some $h \in L\left(\vec{m}^{**}(j)\right)$ such that $W\left(\vec{m}^{**}(A)^h\right) \ge \max_{k\in\{\mathcal{L}|h\}}\left\{W\left(\vec{m}^{**}(A)^k\right)\right\}$, we have that

$$W\left(\vec{m}^{**}(A)\right) \le \sum_{k\in\mathcal{L}}\xi_k\left(\vec{m}^{**}(A)\right)W\left(\vec{m}^{**}(A)^h\right)$$

Using that $\sum_{k\in\mathcal{L}}\xi_k\left(\vec{m}^{**}(A)\right) < \delta r\left(\left|\vec{m}^{**}(A)\right|_k\right)$ for all $k \in \mathcal{L}$ we have that

$$W\left(\vec{m}^{**}(A)\right) < \delta r\left(\left|\vec{m}^{**}(A)\right|_k\right)W\left(\vec{m}^{**}(A)^h\right)$$

for all $k \in \mathcal{L}$, and since $\left|\vec{m}^{**}(A)^h\right|_h = 1$ (since $h \in L\left(\vec{m}^{**}(A)\right)$) and therefore $\vec{m}^{**}(A)^h \in M^*(h)$ we can apply the previous Lemma to get

$$W\left(\vec{m}^{**}(A)\right) < \delta r\left(\left|\vec{m}^{**}(A)\right|_k\right)\max_{l\in\mathcal{L}}\left\{w_l\left(\left|\vec{m}^{**}(A)^h\right|_l\right)\right\}$$

for all $k \in \mathcal{L}$. Using now that $\left|\vec{m}^{**}(A)^h\right|_l = \left|\vec{m}^{**}(A)\right|_l$ for all $l \in \{\mathcal{L}\,|h\}$ and $\left|\vec{m}^{**}(A)^h\right|_h = \left|\vec{m}^{**}(A)\right|_h - 1$ we have

$$\begin{aligned}
W\left(\vec{m}^{**}(A)\right) &< \delta r\left(\left|\vec{m}^{**}(A)\right|_k\right)\max\left\{\max_{l\in\{\mathcal{L}\backslash h\}}\left\{w_l\left(\left|\vec{m}^{**}(A)\right|_l\right)\right\}, w_h\left(\left|\left(\vec{m}^{**}(A)-1\right)\right|_h\right)\right\} \\
&\le \max\left\{\delta r\left(\left|\vec{m}^{**}(A)\right|_k\right)\max_{l\in\{\mathcal{L}\backslash h\}}\left\{w_l\left(\left|\vec{m}^{**}(A)\right|_l\right)\right\}, \delta r\left(\left|\vec{m}^{**}(A)\right|_k\right)w_h\left(\left|\left(\vec{m}^{**}(A)-1\right)\right|_h\right)\right\} \\
&\le \max\left\{\max_{l\in\{\mathcal{L}\backslash h\}}\left\{w_l\left(\left|\vec{m}^{**}(A)\right|_l\right)\right\}, w_h\left(\left|\left(\vec{m}^{**}(A)\right)\right|_h\right)\right\} = \left\{\max_{l\in\mathcal{L}}\left\{w_l\left(\left|\vec{m}^{**}(A)\right|_l\right)\right\}\right\}
\end{aligned}$$

If there is some $h \notin L(\vec{m}^{**}(j))$ such that $W\left(\vec{m}^{**}(A)^h\right) \geq \max_{k\in\{\mathcal{L}\setminus h\}}\left\{W\left(\vec{m}^{**}(A)^k\right)\right\}$ we have that

$$W(\vec{m}^{**}(A)) \leq \delta r(|\vec{m}^{**}(A)|_k) W\left(\vec{m}^{**}(A)^h\right)$$

and since $\vec{m}^{**}(A)^h \in M^{**}(j)$ we have two options. Either there is only one remaining follower in which case we are trivially done or there is more than one follower, in which case we can repeat again the recursion until we reach (39). ■

Lemma A.8 (Induction Step)**.** *Consider any state $\vec{m}$ such that $|\vec{m}|_j \geq 2$ for all $j \in \mathcal{L}$. If*

$$W\left(\vec{m}^j\right) \leq \max\left\{\max_{h\in\{\mathcal{L}|j\}}\{w_h(|\vec{m}|_h)\}, w_j\left(|\vec{m}|_j - 1\right)\right\} \tag{40}$$

for all $j \in \mathcal{L}$, then

$$W(\vec{m}) \leq \max_{j\in\mathcal{L}}\left\{w_j\left(|\vec{m}|_j\right)\right\} \tag{41}$$

Proof of Lemma A.8. By Lemma A.5,

$$W(\vec{m}) \leq \max_{j\in\mathcal{L}}\left\{\delta r\left(|\vec{m}|_j\right) W\left(\vec{m}^j\right)\right\} \tag{42}$$

Using (40), and then noting that $w_j(|\vec{m}|_j) = \delta r\left(|\vec{m}|_j\right) w_j\left(|\vec{m}|_j - 1\right)$ for $j \in \mathcal{L}$, and substituting, (42) becomes

$$W(\vec{m}) \leq \max\left\{\max_{h\in\{\mathcal{L}\setminus j\}}\left\{\delta r\left(|\vec{m}|_j\right) w_h(|\vec{m}|_h)\right\}, w_j\left(|\vec{m}|_j\right)\right\}$$

which implies the result. ■

Proof of Proposition 4.7. Let

$$\Pi^*(m) \equiv v + \left[\sum_{l=1}^{m}\left(1 - \delta\prod_{k=1}^{l} r(k)\right)\right] w \quad \text{for } m = 1, \ldots, q \tag{43}$$

In the proof of Proposition 3.1 we showed that the condition (7) for non-negative surplus in state m in a FTE is $s(m) = \delta^{m-1}(1-\delta)\Pi^*(m) \geq 0 \Leftrightarrow \Pi^*(m) \geq 0$. Note that when $w < 0$, for any $m, m' \leq q$ s.t. $m < m'$, $\Pi^*(m') \geq 0$ implies

$$v \geq \left[\sum_{l=1}^{m'}\left(1 - \delta\prod_{k=1}^{l} r(k)\right)\right](-w) > \left[\sum_{l=1}^{m}\left(1 - \delta\prod_{k=1}^{l} r(k)\right)\right](-w),$$

so that $\Pi^*(m) > \Pi^*(m') \geq 0$. Thus, in particular, $\Pi^*(q) \geq 0 \Rightarrow \Pi^*(m) > 0$ for all $m < q$. It follows that $\Pi^*(q) \geq 0$ is a necessary and sufficient condition for existence of a FTE.

To consider all equilibria, we split the analysis in two cases: (1) $\Pi^*(q) \geq 0$, and (2) $\Pi^*(q) < 0$.

Case 1: $\Pi^*(q) \geq 0$. We focus on two subcases: $\Pi^*(q-1) + w > 0$ and $\Pi^*(q-1) + w \leq 0$. In the first subcase, if the proposed equilibrium were to involve no trade with positive probability at some state m, then L would have a profitable deviation increasing the probability of trade. Hence, the only equilibrium is a FTE. In the second case, if there is no trade with positive probability at some state m, deviations will not always be profitable. In this case we are going to show that if there is mixing at some state, there is some other state with no trade with probability 1.

Case 1.1: $\Pi^*(q-1)+w>0$. To show uniqueness we need to rule out equilibria in which there is no trade with positive probability in some state. First note that since $\Pi^*(1)>0$, in every MPE we must have trade w.p. one at $m=1$, for otherwise there is a sufficiently small $\epsilon>0$ such that the leader could gain by raising her offer by $\epsilon>0$, and having it be accepted for sure.

Now suppose that there is a MPE in which, for some state $m^*\in\{2,\ldots,q-1\}$, we have (i) $\gamma_m\alpha_m=1$ for all $m<m^*$, and (ii) $0<\gamma_{m^*}\alpha_{m^*}<1$ (we study the case $m^*=q$ separately).

Because of mixing at m^*, we must have $s(m^*)=0$, and thus

$$v(m^*)=v(m^*-1)+w_{out}(m^*-1)-w(m^*). \tag{44}$$

At the same time, the value for the leader is $v(m^*)=\gamma_{m^*}\alpha_{m^*}[\delta v(m^*-1)-p(m^*)]+(1-\gamma_{m^*}\alpha_{m^*})\delta v(m^*)$. Solving for $v(m^*)$ and substituting $p(m^*)=\delta\left(w(m^*)-w^{out}(m^*-1)\right)$, we have

$$v(m^*)=\frac{\gamma_{m^*}\alpha_{m^*}\delta}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})}\left(v(m^*-1)+w^{out}(m^*-1)-w(m^*)\right) \tag{45}$$

From (44) and (45), it follows that $v(m^*)=0$.

For L to make an offer in state m^*+1, we need $\delta v(m^*)-p(m^*+1)\geq\delta v(m^*+1)$. Given $v(m^*)=0$, this becomes $w_{out}(m^*)\geq w(m^*+1)$, and by assumption on the equilibrium (trading with certainty in all $m<m^*$ and mixing at m^*), this condition is equivalent to

$$\frac{\delta\gamma_{m^*}\alpha_{m^*}}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})}w_{out}(m^*-1)\geq H_{m^*+1}H_{m^*}\delta^2 w(m^*-1)$$

$$\frac{\gamma_{m^*}\alpha_{m^*}}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})}\delta^{m^*-1}w\geq H_{m^*+1}H_{m^*}\delta\left(\prod_{k=1}^{m^*-1}r(k)\right)\delta^{m^*-1}w$$

$$\frac{\gamma_{m^*}\alpha_{m^*}}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})}\leq H_{m^*+1}H_{m^*}\delta\left(\prod_{k=1}^{m^*-1}r(k)\right)$$

$$\frac{\gamma_{m^*}\alpha_{m^*}}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})}\leq H_{m^*+1}\frac{r(m^*)\gamma_{m^*}\alpha_{m^*}}{1-r(m^*)\delta(1-\gamma_{m^*}\alpha_{m^*})}\delta\left(\prod_{k=1}^{m^*-1}r(k)\right)$$

If $H_{m^*+1}=0$ then $\gamma_{m^*}\alpha_{m^*}=0$ and if $H_{m^*+1}\neq 0$ then we must have that

$$\frac{\gamma_{m^*}\alpha_{m^*}}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})}\leq\frac{r(m^*)\gamma_{m^*}\alpha_{m^*}}{1-r(m^*)\delta(1-\gamma_{m^*}\alpha_{m^*})}$$

but since $r(m^*)<1$ this is only possible if $\gamma_{m^*}\alpha_{m^*}=0$, a contradiction with our initial assumption that $0<\gamma_{m^*}\alpha_{m^*}<1$. It follows that in any MPE that is not a FTE there is a $m^*>1$ such that $\gamma_{m^*}\alpha_{m^*}=0$, and for all $m<m^*$ the equilibrium involves trade with certainty. Because trade stops at $m^*>1$ we have

$$w^{out}(m^*)=w(m^*)=v(m^*)=0$$

Consider then this proposed equilibrium. Because $\gamma_{m^*}\alpha_{m^*}=0$, either the leader does not make an offer at m^*, so that $\gamma_{m^*}=0$, or the follower does not accept the leader's offer, $\alpha_{m^*}=0$. Let's focus first in case in which the leader does not make the equilibrium offer: $\gamma_{m^*}=0$. Note that an offer $\widehat{p} = -\delta w^{out}(m^* - 1) + \epsilon$ for $\epsilon > 0$ is always accepted. By deviating and making this offer, the leader gets a payoff $\delta v(m^* - 1) - \widehat{p}$. This is a profitable deviation if

$$\begin{aligned} \delta v(m^* - 1) - \widehat{p} &> \delta v(m^*) = 0 \\ \delta \left(v(m^* - 1) + w^{out}(m^* - 1) \right) &> \epsilon \\ \delta^{m^*} \times (\Pi^*(m^* - 1) + w) &> \epsilon \end{aligned}$$

where the last line follows since there is trade in every $m < m^*$. Recall that $\Pi^*(m)$ is decreasing in m when $w < 0$, so if $\Pi^*(q-1) + w \geq 0$ we must have that $\Pi^*(m^* - 1) + w > 0$ and therefore there is some $\epsilon > 0$ that makes the offer $\widehat{p}$ preferable for L than not making an offer at $m^* < q$. Similar arguments show if there is some $m^* > 1$ such that $\alpha_{m^*} = 0$, when the leader offers $p(m^*) = \delta \left(w(m^*) - w^{out}(m^* - 1) \right)$, there is a profitable deviation by increasing the offer slightly.

To finish this part of the proof let's focus in the case where $m^* = q$. If $0 < \gamma_q \alpha_q < 1$ (there is randomization) we must have $s(q) = v(q) = 0$ and using (11) we get

$$\begin{aligned} v(q-1) + w^{out}(q-1) = w(q) &= H_q \delta w(q-1) \\ \Pi^*(q-1) + w &= H_q \delta \left(\prod_{k=1}^{q-1} r(k) \right) w \end{aligned}$$

but this is not possible since $\Pi^*(q-1)+w > 0$. Then, we need to rule out equilibria with $\gamma_q \alpha_q = 0$ but with trade with probability 1 at all $m < q$. Since there is no trade at q, $v(q) = w(q) = 0$, but in this case L has a profitable deviation. Let L make the following offer $\widehat{p} = -\delta w^{out}(q-1) + \epsilon$ for some $\epsilon > 0$ which is accepted with probability 1. Thus, L obtains

$$\delta v(q-1) - \widehat{p} = \delta v(q-1) + \delta w^{out}(q-1) - \epsilon$$

and the leader is willing to make this offer with probability 1 if

$$\begin{aligned} \delta v(q-1) + \delta w^{out}(q-1) - \epsilon &> \delta v(q) \\ \delta v(q-1) + \delta w^{out}(q-1) &> \epsilon \\ \delta (\Pi^*(q-1) + w) &> \epsilon \end{aligned}$$

and again if $\Pi^*(q-1) + w > 0$, the deviation is profitable

Case 1.2: $\Pi^*(q-1) + w \leq 0 \leq \Pi^*(q)$. Using the same arguments as in Case 1.1 we can rule out any mixed strategy equilibria except for the possibility that $\gamma_{m^*} \alpha_{m^*} = 0$ for some $m^* > 1$. Here $s(m^*) = v(m^*) = w(m^*) = w^{out}(m^*) = 0$, and then $w(m) = w^{out}(m) = v(m) = 0$ for all $m > m^*$.

Case 2: $\Pi^*(q) < 0$. In this case, as we've shown before, there is no FTE. We separate the analysis in two parts: (1) $0 < \Pi^*(1)$ and (2) $\Pi^*(1) \leq 0$. In the first case, we have that there is trade for low states with probability 1 and potential mixing for high states. The second case is more straightforward since the only class of equilibria that is feasible involves mixing in every state. In both cases we are going to show that, if there is mixing, there is some state $m \leq q$ in which there is no trade with probability 1.

Case 2.1: $\Pi^*(q) < 0 < \Pi^*(1)$. We are going to show that the only other class of equilibria involves no trade at some $m^* \geq 1$. If there is no trade at $m = 1$ we are done, so assume that there is trade at $m = 1$.

Suppose first that $\Pi^*(q-1) \leq 0$. Since $\Pi^*(m)$ is decreasing in m, there is some $m^* < q$ such that $\Pi^*(m) < 0$ for all $m \geq m^*$ but $\Pi^*(m) > 0$ for all $m < m^*$. Since $s(m) = \delta^{m-1}(1-\delta)\Pi^*(m) \geq 0$ for all $m < m^*$, in any equilibrium with trade at $m = 1$ we must have $\gamma_m = \alpha_m = 1$ for all $m < m^*$. We will show that in equilibrium there is no trade at m^*. Assume towards a contradiction that there is trade with positive probability at m^*. Now, since $\Pi^*(m^*) < 0$ by hypothesis, it has to be that $\gamma_{m^*}\alpha_{m^*} < 1$. Thus $s(m^*) = 0$, which with (11) implies that $v(m^*) = 0$ and $w_{out}(m^* - 1) = w(m^*)$. The same logic gives that for all $m > m^*$:

$$w_{out}(m-1) = w(m) \tag{46}$$

Note that using the recursive representation of the value functions for $m^* + 1$ we have

$$\left(\frac{\gamma_{m^*}\alpha_{m^*}\delta}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})}\right) w_{out}(m^*-1) = \delta^2 H_{m^*+1} H_{m^*} w(m^*-1)$$

and since for all $m < m^*$ we have trade with probability 1 in every meeting we can use (12) and (13) to get

$$\left(\frac{\gamma_{m^*}\alpha_{m^*}}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})}\right) = \delta H_{m^*+1} H_{m^*}\left[\prod_{k=1}^{m^*-1} r(k)\right]$$

Using

$$H_m \equiv \frac{r(m)\,\gamma_m\alpha_m}{1-r(m)\,\delta(1-\gamma_m\alpha_m)}$$

we have

$$\frac{\gamma_{m^*}\alpha_{m^*}}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})} = \frac{\gamma_{m^*+1}\alpha_{m^*+1}}{1-r(m^*+1)\,\delta(1-\gamma_{m^*+1}\alpha_{m^*+1})}\frac{\gamma_{m^*}\alpha_{m^*}}{1-r(m^*)\,\delta(1-\gamma_{m^*}\alpha_{m^*})}\delta\left[\prod_{k=1}^{m^*} r(k)\right] \tag{47}$$

Note that

$$\frac{\gamma_{m^*+1}\alpha_{m^*+1}}{1-r(m^*+1)\,\delta(1-\gamma_{m^*+1}\alpha_{m^*+1})} \leq 1$$

so we must have

$$\frac{\gamma_{m^*}\alpha_{m^*}}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})} \leq \frac{\gamma_{m^*}\alpha_{m^*}}{1-r(m^*)\,\delta(1-\gamma_{m^*}\alpha_{m^*})}\delta\left[\prod_{k=1}^{m^*} r(k)\right]$$

and if $\gamma_{m^*}\alpha_{m^*} > 0$, then we have

$$\frac{1-r(m^*)}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})} + r(m^*) \leq \delta\left[\prod_{k=1}^{m^*} r(k)\right]$$

and since $r(k) < 1$ we must have

$$\begin{aligned}\frac{1-r(m^*)}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})} + r(m^*) &< r(m^*)\\ \frac{1-r(m^*)}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})} &< 0\end{aligned}$$

which is false so $\gamma_{m^*}\alpha_{m^*} = 0$.

Next suppose instead that $\Pi^*(q-1) > 0$. In this case $\Pi^*(m) > 0$ for all $m < q$, and thus $m^* = q$. Recall that for all $m < q$ we have that there is trade in every meeting. Assume towards a contradiction that there is trade with positive probability at q. Then $s(q) = 0$ and by (11) we have that $v(q) = 0$. Therefore, we must have

$$w(q) = \delta^{q-1}\left(v + \left[\sum_{l=1}^{q-1}\left(1 - \delta\prod_{k=1}^{l} r(k)\right)\right] w + w\right) \tag{48}$$

Recalling that

$$w(q) = H_q \delta w(q-1)$$

and using (12) and the expression for H_q we have

$$w(q) = \frac{r(q)\gamma_q\alpha_q}{1 - r(q)\delta(1-\gamma_q\alpha_q)}\delta\left[\prod_{k=1}^{q-1} r(k)\right]\delta^{q-1}w$$

Substituting in (48) we have

$$v + \left[\sum_{l=1}^{q}\left(1 - \delta\prod_{k=1}^{l} r(k)\right)\right] w = -\frac{1-\delta r(q)}{1 - r(q)\delta(1-\gamma_q\alpha_q)}(1-\gamma_q\alpha_q)\delta\left[\prod_{k=1}^{q} r(k)\right] w$$

Note that the left hand side is equivalent to $\Pi^*(q)$ and is by assumption negative but since $w < 0$, the right hand side is positive, which is a contradiction. Therefore there is no trade at q.

Case 2.2: $\Pi^*(1) \leq 0$. Note that $\Pi^*(1) \leq 0$ implies that $v + w < 0$, so there is always an equilibrium with no trade at any $m \leq q$. Since $w < 0$, implies that $\Pi^*(m)$ is decreasing we have that $\Pi^*(m) < 0$ for all $m > 1$ and the only other class of equilibria involves randomization at any $m > 1$; i.e $\gamma_m\alpha_m < 1$. Again following Case 2.1 we have that the only other class of equilibria involves no trade at some $m^* \geq 1$ and therefore $v(q) = w(q) = 0$. This concludes the proof. ■

Proposition A.9. *Assume that $z \leq 0 < w$, and let*

$$\Pi_z^*(m) \equiv v + m(z - w) + \left[\sum_{l=1}^{m}\left(1 - \delta\left(\prod_{k=1}^{l} r(k)\right)\right)\right] w$$

(1) If $\Pi_z^*(q) \geq 0$ the unique MPE of the game is a FTE.
(2) If $\Pi_z^*(q-1) \geq 0 > \Pi_z^*(q)$, MPE are either "no trading equilibria" (NTE) or entail no trading with positive probability at q, and thus imply $v(q) = 0$. Thus, $\forall\kappa > 0$, if the leader has to pay an entry cost $\kappa > 0$, she doesn't enter.
(3) If $0 \geq \Pi_z^*(q-1) \geq \Pi_z^*(q)$, all MPE are NTE.

Proof. The proof is similar to that of Proposition 4.7 (click here for details). ■

7. Online Appendix B: Extensions

7.1. **Unobservable Trades.** In our model the state (m or $\vec{m}$ in monopoly and competition) is common knowledge. This has a first order effect on equilibrium outcomes. In any meeting between a leader and a follower, the follower's bargaining power is derived from his outside option. And what gives value to the outside option are free riding opportunities. Common knowledge about the state matters because the state affects the value of free riding opportunities, and thus agents' bargaining power.

The effect of the state on agents' bargaining power is potentially absent when past trades are not observable, and it *is* absent in a pooling equilibrium where the leaders' offer convey no information about the state. It follows that introducing non-observability of trades can change equilibrium payoffs (and outcomes) significantly. In fact, as we show below, non-observability of trades increases agents' bargaining power. But this is not the end of the story. The point is that non-observability of trades affects agents' bargaining power both in the monopoly and competitive games. Thus, the key for the comparison of followers' welfare under monopoly and competition is whether non-observability of trades affects agents' bargaining power differentially in competition and monopoly in a way that would upend the results with full observability.

To study this question, we analyze the monopoly and competitive games under the assumption that trades are not observed by non-participants, following Noe and Wang (2004).[24] Following our previous discussion, we focus on pooling FTE, in which leaders make the same offer to all followers, independently of the state. We also assume, as in Noe and Wang (2004), that agents don't learn from calendar time (agents don't update their belief about the state based on the period in which they are approached).

The plan is the following. We begin with some general results in the monopoly game. We show that followers' equilibrium payoffs in the game with non-observable trades is higher than in the benchmark model with observability. We then move to the competitive game and show that here too, non-observability affects equilibrium payoffs. We wrap up the analysis of both games by characterizing payoffs and proving existence of the pooling FTE in a three agent example. We finish by comparing followers' equilibrium payoffs in both games.

Our results offer two main conclusions. First, equilibrium payoffs of the non-observability games are indeed different than in the benchmark game. Second, still in this case, monopoly is preferred to competition. In a nutshell, non-observability of the state does not alter the fact that free riding opportunities are larger under monopoly than under competition.

(i) Monopolistic Leadership with Non-Observability. Let $\rho(\cdot)$ denote followers' beliefs (i.e., $\rho(m)$ indicates the probability that a follower attaches to the leader being m uncommitted followers short of winning.) In a pooling FTE, $\rho(m) = 1/n$ for all m.[25]

[24]The setup of Noe and Wang (2004) differs from ours in two fundamental aspects. First – in the main part of the paper (with n agents) – Noe and Wang consider what is effectively a unanimity rule: a buyer transacts with n sellers, and gets a payoff of one if she buys all n goods, zero otherwise (the goods are perfect complements). Since unanimity gives veto power to each agent, this effectively eliminates free riding (recall that our result holds for all non-unanimous rules). Second, sellers only care about the money they obtain for selling the good. Hence they are in a pure private values case with no externalities.

[25]As we will see below, this is not the case in the competitive game.

Let $E\hat{w}^{out}$ denote a follower's expected continuation value of accepting an offer by the leader, and $E\hat{w}$ a follower's expected continuation value of rejecting this offer. The optimal relevant offer $\hat{p}$ by the leader in any state $m = 1, 2, \dots, (n+1)/2$ satisfies

$$\delta E\hat{w}^{out} + \hat{p} = \delta E\hat{w}, \tag{49}$$

Let $\hat{w}^{out}(m)$ denote the continuation value of a committed follower in state m, as computed by an outside observer who knows m (i.e., computed under the assumption that the follower does not know the state). Similarly, let $\hat{w}(m)$ denote the continuation value of an uncommitted follower in state m, also as computed by an outside observer who knows m. Finally, let $\hat{v}(m)$ be the continuation value of the leader in state m. Note that since the leader observes past trades, $\hat{v}(m)$ is computed assuming that the leader knows the state.

The expected continuation value of a follower who accepts an offer is given by

$$E\hat{w}^{out} \equiv \sum_{m=1}^{\frac{n+1}{2}} \left(\frac{\rho(m)}{\sum_{j=1}^{\frac{n+1}{2}} \rho(j)} \right) \hat{w}^{out}(m-1) \tag{50}$$

This expression incorporates the fact that having traded, the follower knows that he is one of the $\frac{n+1}{2}$ followers to trade with the leader (i.e., that $m = 1, 2, \dots, (m+1)/2$). Note that $\hat{w}^{out}(m)$ is equivalent to $w^{out}(m)$ in the full information case, so

$$E\hat{w}^{out} = \sum_{m=1}^{\frac{n+1}{2}} \left(\frac{2}{n+1} \right) \delta^{m-1} w = \left(\frac{2}{n+1} \right) \left(\frac{1-\delta^{\frac{n+1}{2}}}{1-\delta} \right) w, \tag{51}$$

where we have used the fact that there is trade after every meeting.

Now consider the expected continuation value of a follower who rejects an offer, $E\hat{w}$. By (49), in any state m the leader offers a transfer that gives the follower a continuation value equal to the discounted continuation value of rejecting the offer, $\delta E\hat{w}$, so

$$\hat{w}(m) = \left(\frac{2}{n+2m-1} \right) \delta E\hat{w} + \left(\frac{n+2m-3}{n+2m-1} \right) \delta \hat{w}(m-1) \tag{52}$$

Solving this recursion,[26] we get

$$\hat{w}(m) = \left(\frac{2}{n+2m-1} \right) \left[\left(\frac{\delta}{1-\delta} \right) (1-\delta^m) E\hat{w} + \left(\frac{n-1}{2} \right) \delta^m w \right] \tag{53}$$

[26] Let $\beta(m) = \frac{2}{n+2m-1}$ denote the probability that a follower is chosen to face the leader, and write (52) as $\hat{w}(m) = \beta(m)\delta E\hat{w} + (1-\beta(m))\,\delta\hat{w}(m-1)$. It is easy to see that $\frac{1-\beta(m)}{1-\delta\beta(m)} = r(m)$ as defined in that text and that $\frac{1-\beta(m)}{\beta(m)} = \frac{1}{\beta(m-1)}$. Therefore, defining $x(m) \equiv \hat{w}(m)/\beta(m)$ we have that the difference equation that describes the value function $\hat{w}(m)$ is given by $x(m) = \delta E\hat{w} + \delta x(m-1)$. This is a linear, autonomous, first order difference equation, with general solution $x(m) = \frac{\delta}{1-\delta} E\hat{w} + C\delta^m$ and boundary condition given by $x(1) = \delta E\hat{w} + \frac{n-1}{2}\delta w$. Substituting,

$$x(m) = \left(\frac{\delta}{1-\delta} \right) (1-\delta^m) E\hat{w} + \left(\frac{n-1}{2} \right) \delta^m w,$$

and using the definition of $x(m)$, we obtain (53).

The expected continuation value for a follower of rejecting the offer is given by

$$E\hat{w} = \frac{1}{n}\sum_{m=1}^{\frac{n+1}{2}} \hat{w}(m) + \frac{n-1}{2n} w \tag{54}$$

where we used $\rho(m) = 1/n$. The second part is the probability of not meeting the leader and successfully free riding on all other followers. Substituting (53) in (54), we obtain

$$E\hat{w} = \frac{1 + \sum_{m=1}^{\frac{n+1}{2}} \left(\frac{2\delta^m}{n+2m-1}\right)}{n - \frac{\delta}{1-\delta}\sum_{m=1}^{\frac{n+1}{2}} \left(\frac{2(1-\delta^m)}{n+2m-1}\right)} \left(\frac{n-1}{2}\right) w \tag{55}$$

which can be used to compute $\hat{w}(m)$ substituting in (53).

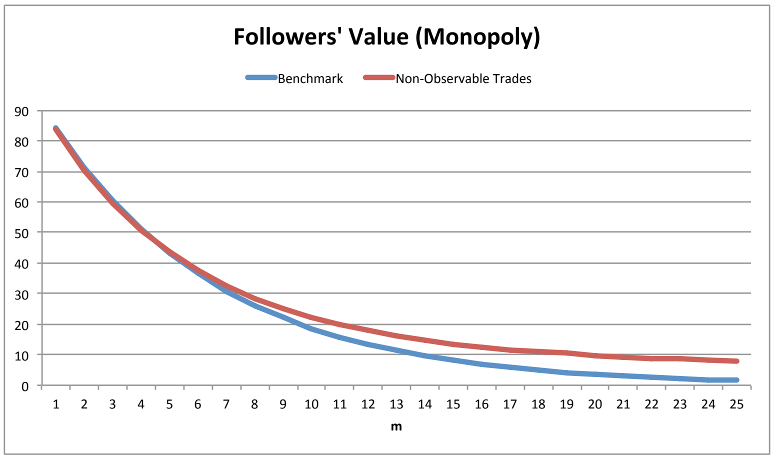


FIGURE B.1. Followers' equilibrium payoff with non-observable trades, $\hat{w}(m)$, and in the benchmark model, $w(m)$ ($\delta = 0.85$, $n = 49$, $w = 100$ and $v = 10$).

Figure B.1 plots $\hat{w}(m)$ together with the followers' payoff in the benchmark model, for $\delta = 0.85$, $n = 49$, $w = 100$ and $v = 10$. Consistent with your intuition, the plot shows that non-observability of trades raises followers' value.

(ii) Existence and Characterization of Pooling FTE with 3 followers [Monopoly]. With $n = 3$, (55), (51) and (53) become

$$E\hat{w} = \left(\frac{6 + \delta(3 + 2\delta)}{18 - \delta(5 + 2\delta)}\right) w, \quad E\hat{w}^{out} = \left(\frac{1+\delta}{2}\right) w$$

and

$$\hat{w}(2) = \left(\frac{2 + 9\delta}{18 - \delta(5 + 2\delta)}\right) \delta w$$

Up to this point, we have proceeded under the assumption that a Pooling FTE exists. We verify this here. Note that the leader is willing to make an offer in state m as long as

$$\delta\hat{v}(m-1) - \hat{p} \geq \delta\hat{v}(m). \tag{56}$$

An equilibrium with trade in every state requires that (56) holds in each state $m = 1, 2$. In order to calculate the expression for (56) we have that the value function for the leader is

$$\hat{v}(m) = \delta\hat{v}(m-1) - \hat{p},$$

where $\hat{p} = \delta[E\hat{w} - E\hat{w}^{out}]$ can be computed from (51) and (55). Again this is a linear, first order, autonomous, difference equation with complete solution given by:

$$\hat{v}(m) = \delta^m v - \frac{1-\delta^m}{1-\delta}\hat{p}$$

and therefore (56) is equivalent to

$$\hat{p} \leq \frac{\delta^m(1-\delta)}{(1-\delta^m)}v \tag{57}$$

Substituting from the expressions for $E\hat{w}$ and $E\hat{w}^{out}$ above, we have

$$\hat{p} = \left(\frac{6+\delta(3+2\delta)}{18-\delta(5+2\delta)} - \frac{1}{2}(1+\delta)\right)w = \frac{1}{2}\left(\frac{11\delta^2+2\delta^3-7\delta-6}{18-\delta(5+2\delta)}\right)w$$

Note that $f(\delta) = 11\delta^2 + 2\delta^3 - 7\delta - 6$ is convex, decreasing for $\delta = 0$ and $f(0) = f(1) = 0$. Therefore, $\hat{p} < 0$. Since $v > 0$, condition (56) holds for $m = 1, 2$ and there is a pooling FTE in the three player case under monopoly.

(iii) Competitive Leadership with Non-Observability. Let $E\hat{W}_\ell^{out}$ denote a follower's expected continuation value of accepting an offer by leader $\ell = A, B$, and $E\hat{W}$ a follower's expected continuation value of rejecting this offer. An optimal relevant offer by $\ell = A, B$ in state $\vec{m}$ verifies

$$\delta E\hat{W}_\ell^{out} + \hat{p}_\ell = \delta E\hat{W}. \tag{58}$$

The expectations

$$E\hat{W}_\ell^{out} = \sum_{m_A=1}^{\frac{n+1}{2}}\sum_{m_B=1}^{\frac{n+1}{2}} \rho(\vec{m})\hat{W}^{out}(\vec{m}^\ell), \quad E\hat{W} = \sum_{m_A=1}^{\frac{n+1}{2}}\sum_{m_B=1}^{\frac{n+1}{2}} \rho(\vec{m})\hat{W}(\vec{m}) \tag{59}$$

are computed with the followers' beliefs ρ. Note that even in a FTE, in the competitive game beliefs over states are not uniform. Since $\rho(\vec{m}) = \Pr(\vec{m} \mid \text{approached } k^{th})\Pr(\text{approached } k^{th})$ and $\Pr(\text{approached } k^{th}) = 1/n$, then[27]

$$\rho(\vec{m}) = \frac{k!}{n}\frac{(\pi_A)^{\left(\frac{n+1}{2}-m_A\right)}(\pi_B)^{\left(k+m_A-\frac{n+1}{2}\right)}}{\left(\frac{n+1}{2}-m_A\right)!\left(k+m_A-\frac{n+1}{2}\right)!}$$

[27]Note that in any state $\vec{m}$ in which follower i is approached in order k we have $m_A + m_B = n + 1 - k$, and therefore $m_B = n + 1 - k - m_A$ and $\frac{n+1}{2} - m_B = k + m_A - \frac{n+1}{2}$.

Note that in a FTE $\hat{W}^{out}(\vec{m})$ is equivalent to $W^{out}(\vec{m})$ as in the benchmark model with observable meetings, which pins down $E\hat{W}_{\ell}^{out}$. Now consider $E\hat{W}$. First, note that $\hat{W}(1,1) = \delta E\hat{W}$. For the boundary states in which $m_A = 1$ and $m_B \geq 2$ (a symmetric expression holds for $(m_A, 1)$, with $m_A \geq 2$), we have

$$\hat{W}(1, m_B) = \frac{\delta E\hat{W}}{m_B} + \frac{m_B - 1}{m_B}\left(\pi_A \delta w_A + \pi_B \delta \hat{W}(1, m_B - 1)\right) \quad \text{for } m_B \geq 2$$

Solving recursively,

$$\hat{W}(1, m_B) = \frac{\delta E\hat{W}(\vec{m})}{m_B} \sum_{j=0}^{m_B-1} (\delta\pi_B)^j + \frac{\delta\pi_A w_A}{m_B} \sum_{j=0}^{m_B-2} (m_B - j - 1)(\delta\pi_B)^j, \tag{60}$$

so that all boundary values again depend on $E\hat{W}(\vec{m})$. Note moreover that for all $\vec{m} \geq (2,2)$ we have

$$\hat{W}(\vec{m}) = \beta'(\vec{m})\, \delta E\hat{W}(\vec{m}) + \left(1 - \beta'(\vec{m})\right)\left(\pi_A \delta \hat{W}(\vec{m}^A) + \pi_B \delta \hat{W}(\vec{m}^B)\right) \tag{61}$$

Thus, *all* values again depend on $E\hat{W}(\vec{m})$. This observation highlights the effect of the non-observability of trades: when the offers are not contingent on the state the leader cannot extract as much surplus from critical followers, who in equilibrium are unaware of their position. This effect is then transmitted recursively to values at the beginning of the game.

(iv) Existence and Characterization of Pooling FTE in a 3-Follower Example [Competition]. To simplify computations, we assume that $w_A = w_B = w$, $\pi_A = \pi_B = 1/2$, and $\underline{v}_A = \underline{v}_B = 0$. With $n = 3$, and given our simplifying assumptions, (60) is

$$\hat{W}(1,2) = \hat{W}(2,1) = \frac{\delta E\hat{W}(\vec{m})}{2}\left(1 + \frac{\delta}{2}\right) + \frac{\delta w}{4}$$

Using (61) we get

$$\hat{W}(2,2) = \frac{1}{3}\delta E\hat{W}(\vec{m}) + \frac{2}{3}\delta\hat{W}(1,2) = \frac{1}{3}\delta\left(E\hat{W}(\vec{m})\left(1 + \delta + \frac{\delta^2}{2}\right) + \frac{\delta w}{2}\right),$$

and substituting in (59)

$$E\hat{W}(\vec{m}) = \frac{\delta}{3}\left[E\hat{W}(\vec{m}) \times \left(\frac{4}{3} + \frac{7\delta}{12} + \frac{\delta^2}{6}\right) + w\left(\frac{1}{4} + \frac{\delta}{6}\right)\right]$$

Therefore,

$$\hat{W}(2,2) = \frac{1}{6}\left(\frac{42-6\delta+\delta^3}{36-\delta\left(16+7\delta+2\delta^2\right)}\right)\delta^2 w$$

and

$$E\hat{W} = \left(\frac{3+2\delta}{36-\delta\left(16+7\delta+2\delta^2\right)}\right)\delta w$$

We now establish existence of this equilibrium. Leader ℓ is willing to make an offer in state m iff $\delta\hat{V}^{\ell}(\vec{m}^{\ell}) - \hat{p}_{\ell} \geq \delta\hat{V}^{\ell}(\vec{m})$. Note that under a FTE, leader ℓ's value function is described by

$$\hat{V}^{\ell}(\vec{m}) = \pi_{\ell}\left(\delta\hat{V}^{\ell}(\vec{m}^{\ell}) - \hat{p}_{\ell}\right) + \pi_j\delta\hat{V}^{\ell}(\vec{m}^{j})$$

and thus ℓ is willing to make an offer in state m iff

$$\delta\hat{V}^{\ell}(\vec{m}^{\ell}) - \hat{p}_{\ell} \geq \frac{\delta\pi_j}{1-\delta\pi_{\ell}}\delta\hat{V}^{\ell}(\vec{m}^{j}) \tag{62}$$

where $\hat{p}_{\ell} = \delta[E\hat{W} - E\hat{W}^{out}_{\ell}]$.

For existence of a pooling FTE we need (62) to hold for all $\vec{m}$, which boils down to

$$\delta\hat{V}^{A}(\vec{m}^{A}) - \hat{p}_{A} \geq \frac{\delta}{2-\delta}\delta\hat{V}^{A}(\vec{m}^{B}) \tag{63}$$

Using that

$$E\hat{W}^{out}(\vec{m}^{A}) = \frac{3+\delta}{6}\delta w$$

we have

$$\hat{p}_A = \delta\left(\frac{3+2\delta}{36-\delta\left(16+7\delta+2\delta^2\right)} - \frac{3+\delta}{6}\right)\delta w < 0$$

Next, note that

$$\hat{V}^A(1,1) = \frac{(\delta\overline{v}_A - \hat{p}_A)}{2}, \quad \hat{V}^A(1,2) = \frac{(\delta\overline{v}_A - \hat{p}_A) + \delta\hat{V}^A(1,1)}{2},$$
$$\hat{V}^A(2,1) = \frac{\left(\delta\hat{V}^A(1,1) - \hat{p}_A\right)}{2}, \textit{and} \quad \hat{V}^A(2,2) = \frac{\left(\delta\hat{V}^A(1,2) - \hat{p}_A\right) + \delta\hat{V}^A(2,1)}{2}$$

Substituting, and noting that $\hat{p}_A \leq 0$, gives that (63) holds for all states $\vec{m} \neq (2,2)$. For $\vec{m} = (2,2)$, condition (63) is equivalent to

$$\frac{\delta^2}{2}\overline{v}_A + \frac{\delta(1-\delta)}{2-\delta}\left(\frac{\delta^2}{2}\overline{v}_A - \left(1+\frac{\delta}{2}\right)\hat{p}_A\right) \geq \hat{p}_A$$

and since $\hat{p}_A \leq 0$, condition (63) also holds for $\vec{m} = (2,2)$.

(v) Comparison of Followers' Equilibrium Payoffs. We are now ready to compare followers' equilibrium payoffs in both games. As in part 4 above, in the competitive game we assume that $w_A = w_B = w$, $\pi_A = \pi_B = 1/2$, and $\underline{v}_A = \underline{v}_B = 0$.

There are two alternative measures that we can consider: (i) the value of the game for the first follower to trade with the leader/s: $\hat{W}(2,2)$ and $\hat{w}(2)$, and (ii) the expected value of the game $E\hat{W}$ and $E\hat{w}$. Using the first measure we have that single leadership is preferred to competition if

$$\frac{42 - 6\delta + \delta^3}{36 - \delta\left(16 + 7\delta + 2\delta^2\right)}\delta < \frac{12 + 54\delta}{18 - \delta(5 + 2\delta)}$$

which reduces to

$$-6\delta\left(230 + 176\delta + 67\delta^2 + 15\delta^3\right) < 432 + \delta^2(5 + 2\delta)\left(42 - 6\delta + \delta^3\right)$$

which is always true.

On the other hand, the comparison in expectation favors monopoly iff

$$\frac{3 + 2\delta}{36 - \delta\left(16 + 7\delta + 2\delta^2\right)}\delta w < \frac{6 + \delta(3 + 2\delta)}{18 - \delta(5 + 2\delta)} \times w$$

which reduces to

$$-(3 + 2\delta)\left(18 + 7\delta + 2\delta^2\right)(1 - \delta)\delta < 6\left[11 + (1 - \delta)\left(25 + 9\delta + 2\delta^2\right)\right]$$

which again is true.

7.2. **Direct Competition.** In our model we assumed a sequential contracting setup, in which leaders and followers make deals in bilateral meetings. An important consequence of this assumption is that competition is "indirect" in the sense that competition affects trades through its effect on followers' outside option.[28]

Here we introduce the possibility of *direct competition*; i.e., that both leaders make simultaneous offers to the followers. To do this we consider a version of our game in which the follower who is selected to negotiate in state $\vec{m}$ meets with leader $j = A, B$ with probability $\pi_j = \pi > 0$, and meets with both leaders with probability $\pi_{AB} = 1 - 2\pi > 0$. This spans the level of direct competition from a situation in which direct competition is unlikely to a case in which it is pervasive. As we will show, our results are quite robust to the presence of direct competition, even for large π_{AB} when there are no frictions (for δ close to one).

Introducing direct competition requires that we **expand the state space** to include the set of principals that can make an offer to the follower at each meeting. Thus we let the *negotiation state* be $\eta \in \{A, B, AB\}$, where η denotes the set of principals that can make an offer to the follower on a meeting at a *step state* $\vec{m}$. The state is then $s = (\vec{m}, \eta)$, and the value functions $W(\vec{m}, \eta)$ and $V(\vec{m}, \eta)$.

The analysis of the game with direct competition introduces additional technical challenges. To attack this problem, we resort to a combination of analytical results and numerical analysis. In particular, we first characterize equilibrium payoffs recursively, expressing equilibrium payoffs in step states $\vec{m}$ as a function of quantities in *forward* step states $(\vec{m}^A, \vec{m}^B)$, imposing only the restriction that strategies are weakly undominated given continuation values.[29] (Click here to see the details of the derivation of the analytical results.) We then use these results to compare equilibrium payoffs under monopoly and competition numerically, for a variety of parameter values.

The gist of the effect of direct competition can be grasped by comparing optimal relevant offers in competitive and non-competitive negotiation states. In a noncompetitive negotiation state, $\eta = A, B$, an optimal relevant offer by leader j satisfies

$$p_j(\vec{m}, j) = -\delta[W(\vec{m}^j) - W(\vec{m})],$$

where $\delta[W(\vec{m}^j) - W(\vec{m})]$ is the follower's discounted gain of going from the current state $\vec{m}$ one step in the direction of leader j. With direct competition, instead, the winning leader j makes an offer

$$p_j(\vec{m}, AB) = \delta\left[(V_\ell(\vec{m}^\ell) - V_\ell(\vec{m}^j)) + (W(\vec{m}^\ell) - W(\vec{m}^j))\right]. \tag{64}$$

[28]This is also how competition among proposers enters in the vast majority of collective bargaining models, including all models in the tradition of Baron and Ferejohn (1989) and Chatterjee et al. (1993).
[29]The negotiation state with competition is similar to a first price auction with two bidders and complete information. As usual in these type of games, there is a continuum of equilibria in weakly dominated strategies in which both bidders bid the same amount, but in all equilibria the winner is the bidder with the highest willingness to pay. As is customary in the literature, we rule out these equilibria by focusing on strategies that are weakly undominated given continuation values, so the common bid is equivalent to the lowest willingness to pay.

This is the joint discounted surplus of the follower and the losing leader ℓ from going from the current state $\vec{m}$ one step in the direction of leader ℓ net of their surplus of going from the current state $\vec{m}$ one step in the direction of leader j.

Equation (64) makes clear the benefit that direct competition can bring to followers: the winning leader has to compensate the follower for the payoff him *and* the losing leader could be making if he were to go in the other direction. The transfer has to be this high to "price out" the competing leader ℓ from the contest. This introduces an additional effect which goes against the free-riding effect that benefits monopoly over competition.

Equation (64) also shows, however, that the potential benefit for a follower in state $\vec{m}$ depends on equilibrium payoffs in states $\vec{m}^\ell$ and $\vec{m}^j$ (having moved one step in ℓ and j's direction, respectively). This is important, because it implies that followers' rent extraction at later states will diminish the potential gain for followers in early states. Thus, whether the rents brought by direct competition can overturn the ranking of monopoly and competition in our benchmark model depends on whether the new rents are large enough and are sufficiently not competed away to overpower the free-riding effect which is still present in this model.

As it turns out, this can fail to happen even for a high probability that the negotiation state is competitive. This can be seen in Tables B.1 - B.5, which provide an extensive characterization of how direct competition affects our results in different environments. In each table, we compute the difference between the equilibrium payoff of an uncommitted follower in the competitive game and monopolistic games for a group size to $n = 21$, for given parameter values $w_A, w_B, \overline{v}_A \overline{v}_B, \delta$ (we fix $\underline{v}_A = \underline{v}_B = 0$). In each column, we fix a value of the probability that principals can make competing offers, π_{AB}. Each row denotes a value of the step state $m = 1, \ldots, 11$. For any given step state m, we compute the difference between the value of an uncommitted follower in the competitive game across the diagonal $W(m, m)$, and the value of an uncommitted follower in the monopolistic game, $w(m)$.

Table B.1. Differential between Followers' Payoffs in Monopoly of Best and Competition: $w_A = 300$, $w_B = 70$, $\overline{v}_A = 200$, $\overline{v}_B = 100$, $\delta = 0.85$, $n = 21$.

	π_{AB}								
m	**0.01**	**0.05**	**0.10**	**0.40**	**0.60**	**0.80**	**0.90**	**0.95**	**0.99**
1	242.1	213.7	189.7	133.3	119.9	112.0	109.1	107.9	107.0
2	128.9	121.9	114.7	81.2	75.1	70.1	67.9	66.8	66.0
3	101.4	96.4	90.9	60.9	52.3	45.1	41.8	40.3	39.1
4	87.2	83.0	78.2	50.6	39.5	30.0	25.8	23.7	22.2
5	77.0	73.5	69.3	44.4	32.5	21.8	16.9	14.6	12.8
6	68.4	65.4	61.8	39.9	28.1	17.2	12.1	9.7	7.7
7	61.4	58.1	55.0	36.1	24.9	14.3	9.3	6.8	4.9
8	54.3	51.5	48.9	32.5	22.2	12.2	7.3	5.0	3.1
9	47.6	45.4	43.2	29.1	19.8	10.5	6.0	3.7	1.9
10	41.6	39.9	38.1	26.0	17.6	9.1	4.9	2.8	1.1
11	**36.2**	**34.9**	**33.4**	**23.1**	**15.7**	**8.0**	**4.1**	**2.1**	**0.6**

In table B.1 we fix $w_A = 300$, $w_B = 70$, $\overline{v}_A = 200$, $\overline{v}_B = 30$, and $\delta = 0.85$. Here A represents a markedly better alternative than B for followers, while at the same time leader A also has a higher willingness to pay for winning than its competitor. The first column reports the results for the case in which direct competition only occurs with probability $\pi_{AB} = 0.01$. The monopoly differential at the beginning of the game (at $m = 11$) is positive (36.2), indicating that a monopoly of the better alternative A dominates competition. The advantage of monopoly increases as we move to states closer to completion, reflecting the higher probability of a win by the worst alternative. Columns 2 and above report the results of similar exercises for larger values of the probability of direct competition. The results show that while the monopoly differential decreases as direct competition becomes more prevalent, for these parameters monopoly dominates competition throughout, even with a probability of direct competition of $\pi_{AB} = 0.99$.

Table B.2. Differential between Followers' Payoffs in Monopoly of Best and Competition: $w_A = 100$, $w_B = 70$, $\overline{v}_A = 200$, $\overline{v}_B = 30$, $\delta = 0.85$, $n = 21$.

	π_{AB}								
m	**0.01**	**0.05**	**0.10**	**0.40**	**0.60**	**0.80**	**0.90**	**0.95**	**0.99**
1	78.4	61.7	47.6	14.4	6.5	1.8	0.1	-0.6	-1.1
2	43.4	37.1	31.3	14.6	9.3	5.9	4.5	3.9	3.5
3	32.6	29.3	26.0	14.7	10.5	7.4	6.2	5.7	5.3
4	27.3	25.3	23.5	14.5	10.5	7.3	5.9	5.2	4.7
5	24.2	22.8	21.3	13.7	9.9	6.5	4.9	4.1	3.5
6	21.6	20.4	19.2	12.8	9.1	5.6	3.9	3.0	2.4
7	19.2	18.3	17.3	11.7	8.2	4.7	3.0	2.2	1.6
8	17.0	16.3	15.5	10.6	7.4	4.1	2.4	1.6	1.0
9	15.0	14.4	13.8	9.6	6.6	3.5	2.0	1.2	0.6
10	13.2	12.7	12.2	8.6	5.9	3.0	1.6	0.9	0.4
11	**11.5**	**11.1**	**10.7**	**7.6**	**5.2**	**2.7**	**1.4**	**0.7**	**0.2**

In table B.2 we reduce the value of a win by the attractive alternative A for the followers, setting $w_A = 100$ (all other parameters stay unchanged). When the probability of direct competition is low enough ($\pi_{AB} \leq 0.9$), monopoly dominates competition for all $m \leq 11$. Differently than in the first table, when the probability of direct competition is sufficiently large ($\pi_{AB} \geq 0.95$) the value of an uncommitted follower is greater in competition than under monopoly for the critical follower $m = 1$. However, even for these high levels of direct competition, the value differential reverts to favoring monopoly at the beginning of the game.

The reason for this is illustrated by the analytical results. The way in which direct competition helps followers is that in competitive negotiation states, the leader needs to pay the follower to dissuade him from favoring the other leader. This rent (eq.5) is increasing in the value differential for the losing leader of going one step towards her direction instead of the winner's direction, and the follower's value differential of going towards the losing leader's direction instead of the winner's. Since the last follower is able to extract the rent from the winner of direct competition at $m = 1$ (A in this case), this reduces the follower's value differential of going towards the loser's direction instead of the winner's at $m = 2$. For the parameters considered here, it is enough to overturn it completely by $m = 2$.

This is crucial to understand why the sole existence of competition is not sufficient to overturn our results. When the races are tight – which is what happens across the diagonal of the competitive game – followers' gains in later step states (say m') reduce leaders' willingness to pay in early step states ($m'' > m'$). This dynamic effect dampens the benefits of direct competition in the initial stages of the game.

Table B.3. Differential between Followers' Payoffs in Monopoly of Best and Competition: $w_A = 100$, $w_B = 70$, $\overline{v}_A = 200$, $\overline{v}_B = 100$, $\delta = 0.85$, $n = 21$.

	π_{AB}								
m	**0.01**	**0.05**	**0.10**	**0.40**	**0.60**	**0.80**	**0.90**	**0.95**	**0.99**
1	74.6	46.2	22.3	-34.2	-47.6	-55.5	-58.4	-59.6	-60.5
2	42.0	31.1	21.1	-7.5	-16.2	-22.1	-24.5	-25.5	-26.3
3	33.5	27.0	20.9	3.0	-3.3	-6.3	-7.0	-7.2	-7.3
4	28.4	24.2	20.3	7.5	3.1	1.0	0.5	0.3	0.3
5	24.7	22.0	19.2	9.5	5.9	3.7	2.8	2.4	2.1
6	21.9	19.9	17.9	10.2	6.9	4.4	3.2	2.5	2.0
7	19.4	17.9	16.4	10.1	7.0	4.3	2.9	2.1	1.5
8	17.1	16.0	14.8	9.6	6.7	3.9	2.4	1.6	1.0
9	15.1	14.3	13.3	8.9	6.2	3.4	2.0	1.2	0.6
10	13.2	12.6	11.9	8.1	5.7	3.0	1.6	0.9	0.4
11	**11.6**	**11.1**	**10.5**	**7.3**	**5.1**	**2.6**	**1.4**	**0.7**	**0.2**

In table B.3 we increase the value of the a win for the principal favoring the unattractive alternative for followers (B) to $\overline{v}_B = 100$, maintaining all other parameters as in Table 2. This is the leader with the lower willingness to pay and who is also least preferred by the followers, so it is the leader that will lose on the competition stage across the diagonal. But because her willingness to pay is now higher, A needs to improve her offer to followers, who derive a larger benefit from direct competition. As expected, this increases the range of π_{AB} for which competition is preferred to monopoly for uncommitted followers at later stages of the game ($m = 1, 2, 3$). However, still monopoly beats competition for all $\pi_{AB} \leq 0.99$ at the initial state. In table B.4 we increase the discount factor from $\delta = 0.85$ to $\delta = 0.95$, maintaining all other parameters unchanged. This increases the range of states for which competition is preferred to monopoly for sufficiently high probability of direct competition. Still in each case the initial value differential favors monopoly.

Table B.4. Differential between Followers' Payoffs in Monopoly of Best and Competition: $w_A = 100$, $w_B = 70$, $\overline{v}_A = 200$, $\overline{v}_B = 100$, $\delta = 0.95$, $n = 21$.

	π_{AB}								
m	**0.01**	**0.05**	**0.10**	**0.40**	**0.60**	**0.80**	**0.90**	**0.95**	**0.99**
1	67.4	11.7	-16.9	-55.7	-61.8	-65.0	-66.1	-66.5	-66.9
2	41.5	15.4	0.6	-25.7	-31.9	-35.9	-37.6	-56.8	-56.8
3	35.5	17.2	6.4	-14.2	-18.2	-19.4	-19.3	-45.4	-43.3
4	32.1	18.6	9.9	-7.1	-9.7	-9.3	-8.2	-35.4	-29.8
5	30.0	19.7	12.4	-2.1	-4.0	-3.4	-2.4	-28.6	-19.6
6	28.9	20.6	14.3	1.5	-0.1	0.0	0.3	-6.6	-11.1
7	28.2	21.3	15.8	4.2	2.5	1.9	1.4	-2.5	-4.5
8	27.6	21.8	17.0	6.3	4.2	2.8	1.8	0.3	0.9
9	27.1	22.1	17.8	7.9	5.4	3.3	1.9	0.7	5.1
10	26.7	22.3	18.5	9.1	6.3	3.5	1.8	0.7	8.4
11	**26.2**	**22.4**	**19.0**	**9.9**	**6.8**	**3.5**	**1.7**	**0.4**	**11.1**

In table B.5 we further reduce the payoff differential between alternatives for the followers, increasing w_B to 90, *and* also increase $\overline{v}_B$ to 200, eliminating the differential in leaders' willingness to pay (We maintain all else constant as in table B.4.) Both of these changes increase the symmetry among alternatives. As the table shows, in this case competition beats monopoly even at the beginning of the game when the probability of direct competition is sufficiently high ($\pi_{AB} \geq 0.90$). In addition, competition is preferred to monopoly for followers at later stages in the game, even for a more moderate probability of direct competition (for $m \leq 7$ at $\pi_{AB} = 0.40$ and $m \leq 4$ at $\pi_{AB} = 0.10$).[30]

Table B.5. Differential between Followers' Payoffs in Monopoly of Best and Competition: $w_A = 100$, $w_B = 90$, $\overline{v}_A = 200$, $\overline{v}_B = 200$, $\delta = 0.95$, $n = 21$.

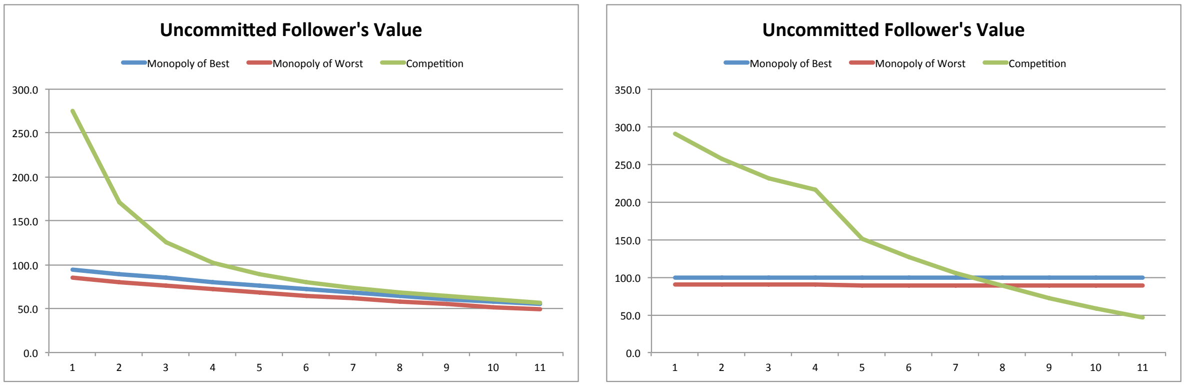
	π_{AB}								
m	**0.01**	**0.05**	**0.10**	**0.40**	**0.60**	**0.80**	**0.90**	**0.95**	**0.99**
1	48.2	-46.8	-95.5	-161.8	-172.1	-177.6	-179.5	-180.2	-180.8
2	28.5	-14.8	-37.3	-70.0	-75.9	-79.3	-80.5	-81.0	-81.4
3	24.7	-4.8	-19.9	-40.0	-41.7	-41.5	-41.0	-40.8	-40.7
4	23.5	1.2	-10.2	-24.0	-24.2	-23.0	-22.4	-22.2	-22.0
5	23.1	5.3	-3.7	-14.1	-13.9	-13.2	-13.1	-13.1	-13.1
6	23.0	8.4	1.0	-7.4	-7.5	-7.7	-8.1	-8.3	-8.6
7	23.0	10.8	4.7	-2.6	-3.3	-4.4	-5.2	-5.7	-6.1
8	23.0	12.8	7.5	0.8	-0.5	-2.3	-3.4	-4.0	-4.5
9	23.0	14.3	9.7	3.4	1.5	-0.9	-2.3	-2.9	-3.4
10	23.0	15.5	11.5	5.3	2.9	0.0	-1.4	-2.1	-2.6
11	**23.0**	**16.4**	**12.9**	**6.8**	**3.9**	**0.7**	**-0.9**	**-1.6**	**-2.1**

Finally, it is interesting to compare followers' welfare in monopoly and competition for δ approaching one (as frictions vanish). In Figure B.2 we do this for the parameters we considered in Table B.5 and a probability of direct competition of $\pi_{AB} = 0.95$, where we showed that competition beats monopoly even at the beginning of the game. The figure plots uncommitted followers equilibrium payoff in monopoly and competition as a function of the step state $m = 1, \ldots, 11$ for $\delta = 0.95$, as in table B.5 (left panel) and $\delta = 0.999$ (right panel). As the figure shows, with the high discount factor the result is reversed, and monopoly again beats competition at the beginning of the game.

Conclusions. Introducing the possibility of direct competition undoubtedly improves the standing of competition vis a vis monopoly. Because in simultaneous bidding both leaders are willing to increase their offers as long as there is a surplus, followers can extract additional rents from the winning leader, who has to raise the transfer enough to exhaust the surplus of the competitor. In a dynamic game, however, the benefit of direct competition is not shared equally among followers. In fact, because these additional rents are heavily captured by followers at the end stages in the game, leaders' willingness to pay to win in direct competition diminishes in early stages, where followers see a reduced advantage from direct competition.

Whether the possibility of direct competition can overturn the ranking of monopoly and competition in our benchmark model depends on whether new rents are large enough and are sufficiently

[30]Still, if given the choice to enter a monopoly or competitive game, followers would still choose monopoly as long as the probability of direct competition is not larger than 0.80.



$\delta = 0.95$ $\delta = 0.999$

FIGURE B.2. Followers' Value in Monopoly and Competition for $\delta = 0.95$ (left panel) and $\delta = 0.999$ (right panel). Parameters as in last column of Table B.5: $w_A = 100$, $w_B = 90$, $\overline{v}_A = 200$, $\overline{v}_B = 200$, $n = 21$, with probability of direct competition $\pi_{AB} = 0.95$.

not competed away to overpower the free-riding effect that we identified in the paper, and which is still present here. In particular, the race between the ability to capture rents in direct competition and the free-riding effect can benefit competition for some parameters. However, as long as both direct competition and bilateral bargaining are possible, the effect of free-riding on bargaining persists, and implies that for many parameter configurations monopoly beats competition even when direct competition is prevalent. We conclude that the mechanism we identify in the paper on the bargaining consequences of free-riding opportunities is still important in the presence of direct competition.

7.3. **Commitment to Reject.** In the paper we assumed that if a follower rejects an offer from a/the leader, the follower returns to the pool of uncommitted followers, and can possibly accept an offer at a later time (if one is extended to him). Here we study an extension of the model in which followers can choose to reject offers permanently, leaving the pool of uncommitted followers (i.e., followers can now accept, reject, or *leave*).

In this context the state space is multidimensional even in the single leader case. The reason is that when a supporter leaves he reduces the pool of uncommitted followers without increasing the support for the leader. Because this reduces the free-riding opportunities of the remaining uncommitted followers, we need to keep track of both the additional number of followers which the leader needs to win, m, and the number of remaining uncommitted followers, u. We also need to consider the possibility that the leader/s knowingly makes a low offer to reduce the number of uncommitted followers.

For simplicity, we consider a three-agent example. We show that when the leader/s value for winning is sufficiently large, the equilibrium payoff of an uncommitted follower in a MPE of the monopoly game is larger than his equilibrium payoff in the competitive game.

Consider first monopoly. From the same arguments that we used in the benchmark model, we know that for large v, the only candidate for a MPE is a full trading equilibrium (FTE). We therefore directly focus on equilibria of this class. We show that when v is large there exists a FTE, and the equilibrium payoff of an uncommitted follower is given by:

$$w(2,3) = (1+2\delta)\,(\delta^2/3)w$$

Proof. Consider first $(m, u) = (1, 1)$, a state in which a single remaining uncommitted follower is pivotal for the decision (after one follower accepted and one follower left). Note that leaving gives the follower a payoff of zero, while rejecting an offer gives him a payoff of $\delta w(1, 1)$. Then an optimal relevant offer satisfies

$$\delta w + p(1,1) = \max\{\delta w(1,1), 0\} \Rightarrow p\,(1,1) = \max\{\delta w(1,1), 0\} - \delta w.$$

Then $w(1, 1) = \max\{\delta w(1, 1), 0\} \Rightarrow w(1, 1) = 0$. It follows that $p\,(1, 1) = -\delta w$, giving the leader a payoff of $v(1, 1) = \delta v - p(1, 1) = \delta(v + w)$. Since $v(1, 1) > 0$, deviating to not making an offer – which gives the leader a payoff of $\delta v(1, 1) < v(1, 1)$ – is not a profitable deviation. And since the follower is pivotal, deviating to a non-relevant offer $\tilde{p}\,(1, 1) < p(1, 1)$ gives her a payoff of zero, and is not profitable either.

Consider next the state in which exactly one follower has committed his support to the leader and nobody left; $(m, u) = (1, 2)$. Note that the optimal relevant offer by the leader must verify

$$p\,(1,2) + \delta w = \delta \max\left\{w_{out}\,(1,1)\,, w\,(1,2)\right\}$$

Suppose first that $w\,(1, 2) \geq w_{out}\,(1, 1) = \delta w$. Then $p\,(1, 2) = \delta w\,(1, 2) - \delta w$, and

$$w(1,2) = \frac{1}{2}\delta w\,(1,2) + \frac{1}{2}\delta w \Rightarrow w(1,2) = \frac{\delta}{2-\delta}w < \delta w = w_{out}\,(1,1)\,,$$

a contradiction. Thus, the follower prefers to leave than to reject the offer, and we must have $w\,(1, 2) \leq w_{out}\,(1, 1) = \delta w$. Thus $p\,(1, 2) = -\delta\,(1 - \delta)\,w$, giving the leader a payoff $v(1, 2) = \delta v - p(1, 2) = \delta(v + (1 - \delta)w) > 0$. Since $v(1, 2) > 0$, deviating to not making an offer – which gives the leader a payoff of $\delta v(1,2) < v(1,2)$ – is not a profitable deviation. In this context, however, the leader could also deviate to making a non-relevant offer $\tilde{p}(1,2) < -\delta(1-\delta)w$ to transition to the state $(1,1)$. In this case she obtains a payoff $\delta v(1,1) = \delta^2(v+w)$. This is not a profitable deviation if $(1-\delta)v + (1-2\delta)w \geq 0$, which is always satisfied for large v. Then

$$w(1,2) = \frac{1}{2}\delta w_{out}(1,1) + \frac{1}{2}\delta w = \frac{1}{2}\delta(1+\delta)w$$

Consider next the state $(m,u) = (2,2)$, reached after the first follower chose to leave. Note that in this situation, both followers are critical (and hence there are no free riding opportunities). Thus leaving gives a follower a payoff of zero. An optimal relevant offer then satisfies

$$p(2,2) + \delta w_{out}(1,1) = 0 \Rightarrow p(2,2) = -\delta^2 w,$$

giving the leader a payoff of $v(2,2) = \delta v(1,1) - p(2,2) = \delta^2(v+2w) > 0$. As before, deviating to not making an offer only delays this outcome and is not a profitable deviation. And since the follower is pivotal, deviating to a non-relevant offer $\tilde{p}(2,2) < p(2,2)$ gives her a payoff of zero, and is not profitable. Therefore $w(2,2) = 0$, and $w_{out}(2,2) = \delta^2 w$.

Finally, consider the initial state $(2,3)$. Note that the optimal relevant offer by the leader must verify

$$p(2,3) + \delta w_{out}(1,2) = \delta \max\{w(2,3), w_{out}(2,2)\}$$

Assume first that $w(2,3) \geq w_{out}(2,2)$. Then $p(2,3) = \delta[w(2,3) - w_{out}(1,2)]$, and

$$w(2,3) = \frac{1}{3}\delta w(2,3) + \frac{2}{3}\delta w(1,2) \Rightarrow w(2,3) = \frac{\delta^2(1+\delta)}{(3-\delta)}w < \delta^2 w = w_{out}(2,2),$$

a contradiction. Thus, the follower prefers to leave than to simply reject the offer, and we must have $w(2,3) \leq w_{out}(2,2) = \delta^2 w$. Thus

$$p(2,3) = \delta w_{out}(2,2) - \delta w_{out}(1,2) = -(1-\delta)\delta^2 w,$$

giving the leader a payoff

$$v(2,3) = \delta v(1,2) - p(2,3) = \delta^2(v + 2(1-\delta)w) > 0.$$

Since $v(2,3) > 0$, deviating to not making an offer – which gives the leader a payoff of $\delta v(2,3) < v(2,3)$ – is not a profitable deviation. In this context, however, the leader could also deviate to making a non-relevant offer $\tilde{p}(2,3) < -(1-\delta)\delta^2 w$ to transition to the state $(2,2)$. In this case she obtains a payoff $\delta v(2,2) = \delta^3(v+2w)$. This is not a profitable deviation if $v(1-\delta)+2(1-2\delta)w \geq 0$, which is always satisfied for large v. Then

$$w(2,3) = \frac{1}{3}\delta^3 w + \frac{2}{3}\delta w(1,2) \Rightarrow w(2,3) = (1+2\delta)(\delta^2/3)w$$

■

We now consider followers' equilibrium payoffs in the competitive game. In this case the state is $(\vec{m}, u)$. As before, we assume that leaders have a high valuation for winning. We also assume that both leaders prefer to retain the status quo rather than losing to the competition; i.e.,

$\underline{v}_A < 0$ and $\underline{v}_B < 0$.[31] Under this conditions, we show that there is a full trading equilibrium, and that the followers' payoffs are given by

$$W(2,2,3) = \left(\frac{1+2\delta}{3}\right)\left[(\delta\pi_A)^2 w_A + (\delta\pi_B)^2 w_B\right].$$

Proof. Consider first the state $(\vec{m}, u) = (1, 1, 1)$, in which there is only one uncommitted follower, and both leaders need the support of only one additional follower to win. Suppose the follower meets with leader $\ell = A, B$, who makes him an offer. Note that leaving gives the follower a payoff of zero, while rejecting the offer gives him a payoff of $\delta W(1, 1, 1)$. Then an optimal relevant offer by ℓ satisfies

$$\delta w + p_\ell(1,1,1) = \max\{\delta W(1,1,1), 0\} \Rightarrow p_\ell(1,1,1) = \max\{\delta W(1,1,1), 0\} - \delta w,$$

Since this is the same for both leaders, we have that $W(1, 1, 1) = \max\{\delta W(1, 1, 1), 0\} \Rightarrow W(1, 1, 1) = 0$. It follows that $p_\ell(1, 1, 1) = -\delta w$, giving the leader a payoff of $V_\ell^\ell(1, 1, 1) = \delta\overline{v}_\ell - p_\ell(1, 1, 1) = \delta(\overline{v}_\ell + w) > 0$. Note that $\ell's$ payoff in state $(\vec{m}, u) = (1, 1, 1)$ after leader $j \neq \ell$ meets with the follower is $V_\ell^j(1, 1, 1) = \underline{v}_\ell < V_\ell^\ell(1, 1, 1)$. Thus, deviating to not making an offer gives the leader a payoff of

$$\delta V_\ell(1,1,1) = \delta[\pi_\ell V_\ell^\ell(1,1,1) + \pi_j V_\ell^j(1,1,1)] < V_\ell^\ell(1,1,1)$$

and is not profitable. Now consider a non-relevant offer, $\tilde{p}_\ell(1, 1, 1) < p_\ell(1, 1, 1)$. Since the follower is indifferent between rejecting or leaving, we have to consider two possibilities. If he leaves, the leader gets a payoff of zero. If he rejects, the leader gets a payoff of $\delta V_\ell(1, 1, 1)$. In both cases, this gives the leader a lower payoff than what she obtains in equilibrium, and thus a non-relevant offer is not a profitable deviation. To summarize, we have

$$\begin{aligned} W(1,1,1) &= 0 \\ V_A(1,1,1) &= \delta\pi_A(\overline{v}_A + w_A) + \delta\pi_B\underline{v}_A \\ V_B(1,1,1) &= \delta\pi_A\underline{v}_B + \delta\pi_B(\overline{v}_B + w_B) \\ W_{out}(1,1,1) &= \delta(\pi_A w_A + \pi_B w_B) \end{aligned}$$

Consider next the state $(\vec{m}, u) = (1, 2, 1)$. In this case there is one uncommitted follower, but B is two steps away from winning. It follows that by getting the follower's support, B can only assure that the status quo will prevail, but can not win. In this case leaving gives the follower a payoff of zero, so the best outside option is to reject, giving a payoff of $\delta W(1, 2, 1)$. Thus the optimal relevant offers satisfy

$$\begin{aligned} p_B(1,2,1) &= \delta W(1,2,1) \\ p_A(1,2,1) + \delta w_A &= \delta W(1,2,1) \end{aligned}$$

[31]The assumption that leaders have a high valuation for winning implies that in equilibrium meetings result in trades in all but two states. The exception is the state $(\vec{m}, u) = (1, 2, 1)$ – and symmetrically, the state $(\vec{m}, u) = (2, 1, 1)$ – in which leader B cannot win (she needs two additional supporters, but there is only one uncommitted follower) but can force a tie. Here equilibrium behavior depends on whether B prefers to retain the status quo rather than losing to A, or vice versa. For concreteness, we assume that both leaders prefer to retain the status quo rather than losing to the competition, i.e., $\underline{v}_A < 0$ and $\underline{v}_B < 0$. We then show that our main result is unchanged if leaders prefer the status quo to losing to the competition.

Leader $\ell = A, B$ prefers to make this offer instead of passing iff

$$\begin{aligned} 0 - p_B(1,2,1) &\geq \delta V_B(1,2,1) \\ \delta \overline{v}_A - p_A(1,2,1) &\geq \delta V_A(1,2,1) \end{aligned}$$

or equivalently

$$\begin{aligned} V_B(1,2,1) + W(1,2,1) &\leq 0 \\ V_A(1,2,1) + W(1,2,1) &\leq \overline{v}_A + w_A \end{aligned}$$

Note that in equilibrium

$$\begin{aligned} W(1,2,1) &= \pi_A \delta W(1,2,1) + \pi_B \delta W(1,2,1) = 0 \\ V_A(1,2,1) &= \pi_A(\delta \overline{v}_A - p_A(1,2,1)) = \pi_A \delta(\overline{v}_A + w_A) \\ V_B(1,2,1) &= \pi_A \delta \underline{v}_B \end{aligned}$$

Thus substituting,

$$\begin{aligned} \pi_A \delta \underline{v}_B &\leq 0 \\ \overline{v}_A(1 - \pi_A \delta) + w_A(1 - \pi_A \delta) &\geq 0 \end{aligned}$$

Note that the first inequality is satisfied iff $\underline{v}_B \leq 0$, while the second is satisfied for large $\overline{v}_A$. Thus, leaders prefer to make relevant offers than to pass. Note moreover that if leader ℓ deviates to a non-relevant offer, she gets $\delta V_\ell(1,2,0) = 0$, which is not a profitable deviation. Analogously, in state $(\vec{m}, u) = (2,1,1)$, we have

$$\begin{aligned} W(2,1,1) &= 0 \\ V_B(2,1,1) &= \pi_B \delta(\overline{v}_B + w_B) \\ V_A(2,1,1) &= \pi_B \delta \underline{v}_A \end{aligned}$$

Next, consider the state $(\vec{m}, u) = (1,2,2)$, where two followers remain uncommitted, A is one step from winning, and B is two steps from winning. Optimal relevant offers verify

$$\begin{aligned} p_A(1,2,2) + \delta w_A &= \delta \max\{W_{out}(1,2,1), W(1,2,2)\} \\ p_B(1,2,2) + \delta W_{out}(1,1,1) &= \delta \max\{W_{out}(1,2,1), W(1,2,2)\} \end{aligned}$$

Assume first that the equilibrium is such that $W(1,2,2) \geq W_{out}(1,2,1)$ which implies that

$$W(1,2,2) = (\delta/2) W(1,2,2) + \frac{1}{2}\pi_A \delta w_A + \frac{1}{2}\pi_B \delta W(1,1,1) \Rightarrow W(1,2,2) = \frac{\pi_A \delta}{2 - \delta} w_A.$$

Since we assume that $W(1,2,2) \geq W_{out}(1,2,1)$ it must be that

$$\frac{\pi_A \delta}{2 - \delta} w_A \geq \pi_A \delta w_A + \pi_B \delta W_{out}(1,1,0) = \pi_A \delta w_A \Rightarrow \delta \geq 1,$$

which is a contradiction. Therefore $W(1,2,2) < W_{out}(1,2,1)$, so that leaving is the relevant outside option. The offers are then given by

$$\begin{aligned} p_A(1,2,2) &= -\delta\left(\frac{1-\delta}{1-\delta\pi_B}\right) w_A \\ p_B(1,2,2) &= -\delta^2 \pi_B \left(w_B - \frac{\delta \pi_A}{1 - \delta \pi_B} w_A\right) \end{aligned}$$

Equilibrium payoffs are therefore

$$\begin{aligned} W(1,2,2) &= \frac{1}{2}\delta W_{out}(1,2,1) + \frac{1}{2}\left(\pi_A \delta w_A + \pi_B \delta W(1,1,1)\right) \\ W_{out}(1,2,2) &= \pi_A \delta w_A + \pi_B \delta W_{out}(1,1,1) \\ V_A(1,2,2) &= \pi_A\left(\delta \overline{v}_A - p_A(1,2,2)\right) + \pi_B \delta V_A(1,1,1) \\ V_B(1,2,2) &= \pi_A \delta \underline{v}_B + \pi_B\left(\delta V_B(1,1,1) - p_B(1,2,2)\right) \end{aligned}$$

or substituting,

$$\begin{aligned} W(1,2,2) &= \frac{1}{2}\pi_A \delta w_A (1+\delta) \\ W_{out}(1,2,2) &= \pi_A \delta w_A + \pi_B \delta^2[\pi_A w_A + \pi_B w_B] \\ V_A(1,2,2) &= \pi_A \delta\left(\overline{v}_A + \left(\frac{1-\delta}{1-\delta\pi_B}\right) w_A\right) + \pi_B \delta^2[\pi_A(\overline{v}_A + w_A) + \pi_B \underline{v}_A] \\ V_B(1,2,2) &= \pi_A \delta \underline{v}_B + \pi_B \delta^2\left([\pi_A \underline{v}_B + \pi_B(\overline{v}_B + w_B)] + \pi_B\left(w_B - \frac{\delta\pi_A}{1-\delta\pi_B} w_A\right)\right) \end{aligned}$$

Each leader prefers to make a relevant offer than to pass iff

$$\begin{aligned} \delta \overline{v}^A - p_A(1,2,2) &\geq \delta V_A(1,2,2) \\ \delta V_B(1,1,1) - p_B(1,2,2) &\geq \delta V_B(1,2,2) \end{aligned}$$

Substituting, these are

$$\begin{aligned} (1 - \delta\pi_A(1+\delta\pi_B))\overline{v}_A &\geq A \\ (1-\delta\pi_B)\overline{v}_B &\geq B \end{aligned}$$

for some constants A and B that do not depend on $\overline{v}_A, \overline{v}_B$. Now $1 > \delta\pi_A(1+\delta\pi_B)$ iff $\delta\pi_A[1 + \delta(1-\pi_A)] < 1$. LHS is increasing in δ, so it is enough to show that $\pi_A[2-\pi_A] \leq 1$. But the LHS of this expression is maximized at $\pi_A = 1$, attaining a value of 1. It follows that $\delta\pi_A(1+\delta\pi_B) < 1$, and hence that both inequalities are satisfied for large $\overline{v}_A, \overline{v}_B$.

Moreover, each leader prefers to make a relevant offer than to make a non-relevant offer iff

$$\begin{aligned} \delta \overline{v}^A - p_A(1,2,2) &\geq \delta V_A(1,2,1) \\ \delta V_B(1,1,1) - p_B(1,2,2) &\geq \delta V_B(1,2,1) \end{aligned}$$

or substituting,

$$\begin{aligned} \overline{v}^A(1-\pi_A\delta) + \left(\frac{1-\delta}{1-\delta\pi_B}\right) w_A &\geq \pi_A \delta w_A \\ (\overline{v}_B + w_B) + \left(w_B - \frac{\delta\pi_A}{1-\delta\pi_B} w_A\right) &\geq 0 \end{aligned}$$

which again hold for high $\overline{v}_A, \overline{v}_B$, and we are done.

Analogously,

$$\begin{aligned}
W(2,1,2) &= \frac{1}{2}\pi_B\delta w_B(1+\delta) \\
W_{out}(2,1,2) &= \pi_B\delta w_B + \pi_A\delta^2[\pi_A w_A + \pi_B w_B] \\
V_B(2,1,2) &= \pi_B\delta\left(\overline{v}_B + \left(\frac{1-\delta}{1-\delta\pi_A}\right)w_B\right) + \pi_A\delta^2[\pi_B(\overline{v}_B + w_B) + \pi_A\underline{v}_B] \\
V_A(2,1,2) &= \pi_B\delta\underline{v}_A + \pi_A\delta^2\left([\pi_B\underline{v}_A + \pi_A(\overline{v}_A + w_A)] + \pi_A\left(w_A - \frac{\delta\pi_B}{1-\delta\pi_A}w_B\right)\right)
\end{aligned}$$

Consider next the state $(2,2,2)$, where there are two uncommitted followers and both leaders are still two steps away from winning. Since leaving gives a follower a payoff of $\delta W_{out}(2,2,1) = 0$, the relevant outside option is given by $\delta W(2,2,2)$ which implies that optimal relevant offers will satisfy

$$\begin{aligned}
p_A(2,2,2) + \delta W_{out}(1,2,1) &= \delta W(2,2,2) \\
p_B(2,2,2) + \delta W_{out}(2,1,1) &= \delta W(2,2,2)
\end{aligned}$$

It follows that

$$W(2,2,2) = \frac{1}{2}\delta W(2,2,2) + \frac{1}{2}\delta(\pi_A W(1,2,1) + \pi_B W(2,1,1))$$

and using $W(1,2,1) = W(2,1,1) = 0$, we have $W(2,2,2) = 0$, so that

$$\begin{aligned}
p_A(2,2,2) &= -\delta W_{out}(1,2,1) \\
p_B(2,2,2) &= -\delta W_{out}(2,1,1)
\end{aligned}$$

and equilibrium payoffs are given by

$$\begin{aligned}
W(2,2,2) &= 0 \\
W_{out}(2,2,2) &= \delta(\pi_A W_{out}(1,2,1) + \pi_B W_{out}(2,1,1)) \\
V_A(2,2,2) &= \pi_A(\delta V_A(1,2,1) - p_A(2,2,2)) + \pi_B\delta V_A(2,1,1) \\
V_B(2,2,2) &= \pi_A\delta V_B(1,2,1) + \pi_B(\delta V_B(2,1,1) - p_B(2,2,2))
\end{aligned}$$

which after substituting, become

$$\begin{aligned}
W(2,2,2) &= 0 \\
W_{out}(2,2,2) &= (\delta\pi_A)^2 w_A + (\delta\pi_B)^2 w_B \\
V_A(2,2,2) &= (\delta\pi_A)^2(\overline{v}_A + w_A) + (\delta\pi_A)^2 w_A + (\delta\pi_B)^2\underline{v}_A \\
V_B(2,2,2) &= (\delta\pi_B)^2(\overline{v}_B + w_B) + (\delta\pi_B)^2 w_B + (\delta\pi_A)^2\underline{v}_B
\end{aligned}$$

Leaders prefer to make these offers to passing iff

$$\begin{aligned}
\delta V^A(1,2,1) - p_A(2,2,2) &\geq \delta V_A(2,2,2) \\
\delta V^B(2,1,1) - p_B(2,2,2) &\geq \delta V_B(2,2,2)
\end{aligned}$$

Substituting, these are

$$\begin{aligned}
\delta^2(1-\delta\pi_A)\pi_A(\overline{v}_A + w_A) - \pi_B\delta^2\pi_B\delta\underline{v}_A + \delta(1-\delta\pi_A)W_{out}(1,2,1) &\geq 0 \\
\delta^2(1-\delta\pi_B)\pi_B(\overline{v}_B + w_B) - \pi_A\delta^2\pi_A\delta\underline{v}_B + \delta(1-\delta\pi_B)W_{out}(2,1,1) &\geq 0
\end{aligned}$$

which always hold for high $\overline{v}_A, \overline{v}_B$. Similarly, it is easy to check that a deviation to making a non-relevant offer is not profitable either.

Finally, consider the initial state $(\vec{m}, u) = (2,2,3)$. First note that the offers must verify

$$\begin{aligned} p_A(2,2,3) + \delta W_{out}(1,2,2) &= \delta \max\{W(2,2,3), W_{out}(2,2,2)\} \\ p_B(2,2,3) + \delta W_{out}(2,1,2) &= \delta \max\{W(2,2,3), W_{out}(2,2,2)\} \end{aligned}$$

Assume first that $W(2,2,3) \geq W_{out}(2,2,2)$. Then

$$W(2,2,3) = \frac{1}{3}\delta W(2,2,3) + \frac{2}{3}(\pi_A \delta W(1,2,2) + \pi_B \delta W(2,1,2)),$$

and substituting,

$$W(2,2,3) = \frac{1}{(3-\delta)}\big((\delta\pi_A)^2 w_A(1+\delta) + (\delta\pi_B)^2 w_B(1+\delta)\big)$$

Since we have assumed that $W(2,2,3) \geq W_{out}(2,2,2)$, we need to verify that

$$\frac{1}{(3-\delta)}\big((\delta\pi_A)^2 w_A(1+\delta) + (\delta\pi_B)^2 w_B(1+\delta)\big) \geq (\delta\pi_A)^2 w_A + (\delta\pi_B)^2 w_B$$

which is false. Therefore, we must have that $W(2,2,3) \leq W_{out}(2,2,2)$ and the optimal relevant offers are

$$\begin{aligned} p_A(2,2,3) &= \delta[W_{out}(2,2,2) - W_{out}(1,2,2)] \\ p_B(2,2,3) &= \delta[W_{out}(2,2,2) - W_{out}(2,1,2)] \end{aligned}$$

The leaders prefer to make these offers to passing iff

$$\begin{aligned} \delta V_A(1,2,2) - p_A(2,2,3) &\geq \delta V_A(2,2,3) \\ \delta V_B(2,1,2) - p_B(2,2,3) &\geq \delta V_B(2,2,3) \end{aligned}$$

or substituting, iff

$$\begin{aligned} \overline{v}_A \pi_A[(1-\delta\pi_A)(1+\delta\pi_B) - (\delta)^2\pi_B\pi_A] &\geq A \\ \overline{v}_B \pi_B[(1-\delta\pi_B)(1+\delta\pi_A) - (\delta)^2\pi_A\pi_B] &\geq B \end{aligned}$$

where A and B are constants that do not depend on $\overline{v}_A, \overline{v}_B$. Now, $(1-\delta\pi_A)(1+\delta\pi_B) - (\delta)^2\pi_B\pi_A \geq 0$ iff $1+\delta \geq 2\delta\pi_A(1+\delta(1-\pi_A))$. The RHS of this inequality is maximized for a value of $\pi_A = (1+\delta)/2\delta$. Substituting, the previous inequality becomes $\delta \leq 1$. It follows that leaders prefer to make a relevant offer to passing for high $\overline{v}_A, \overline{v}_B$.

Similarly, the leaders prefer making relevant offers to making not relevant offers iff

$$\begin{aligned} \delta V_A(1,2,2) - p_A(2,2,3) &\geq \delta V_A(2,2,2) \\ \delta V_B(2,1,2) - p_B(2,2,3) &\geq \delta V_B(2,2,2) \end{aligned}$$

Substituting, we have

$$\begin{aligned} \overline{v}_A \delta\pi_A[1+\delta(1-2\pi_A)] &\geq A \\ \overline{v}_B \delta\pi_B[1+\delta(1-2\pi_B)] &\geq B \end{aligned}$$

where A and B are constants that do not depend on $\overline{v}_A, \overline{v}_B$. Since $1+\delta(1-2\pi_\ell) \geq 0$ for all $\delta, \pi_\ell \in (0,1)$, these inequalities always hold for high $\overline{v}_A, \overline{v}_B$, and we are done. The value for the uncommitted follower is then

$$W(2,2,3) = \left(\frac{1+2\delta}{3}\right)\left[(\delta\pi_A)^2 w_A + (\delta\pi_B)^2 w_B\right]$$

■

We can now compare followers values in monopoly and competition. In monopoly

$$w(2,3) = (1+2\delta)\,(\delta^2/3)w$$

and in competition

$$W(2,2,3) = \left(\frac{1+2\delta}{3}\right)\left[(\delta\pi_A)^2 w_A + (\delta\pi_B)^2 w_B\right]$$

Let $\max\{w_A, w_B\} = w$. A monopoly of the best alternative is preferred to competition for all $w_A, w_B > 0$ iff

$$(\pi_A)^2 + (\pi_B)^2 \leq 1$$

which is always true.

In the comparison above we maintained the assumption that leaders have a high valuation for winning. We also assumed that both leaders prefer to retain the status quo rather than losing to the competition; i.e., $\underline{v}_A < 0$ and $\underline{v}_B < 0$. These assumptions imply that while in state $(\vec{m}, u) = (1,2,1)$ B cannot win (B needs two additional supporters, but there is only one uncommitted follower), it still makes a relevant offers in order to force a tie. Similarly, A makes a relevant offer in state $(\vec{m}, u) = (2,1,1)$. The main result, however, doesn't depend on this assumption. Proceeding similarly as above we can show that if instead leaders prefer the status quo to losing to the competition, followers value is given by

$$W(2,2,3) = \frac{1}{3}\delta^2\left\{\left(\frac{1+\delta(1+\pi_A)}{1-\delta\pi_B}\right)(\pi_A)^2 w_A + \left(\frac{1+\delta(1+\pi_B)}{1-\delta\pi_A}\right)(\pi_B)^2 w_B\right\}$$

It follows that a monopoly of the best alternative is preferred to competition for all $w_A, w_B > 0$ iff

$$(1+2\delta) \geq \left(\frac{1+\delta(1+\pi_A)}{1-\delta\pi_B}\right)(\pi_A)^2 + \left(\frac{1+\delta(1+\pi_B)}{1-\delta\pi_A}\right)(\pi_B)^2$$

or equivalently,

$$\left(\frac{1+\delta(1+\pi_A)}{(1+2\delta)}\right)\frac{1}{1-\delta\pi_B}(\pi_A)^2 + \left(\frac{1+\delta(1+\pi_B)}{(1+2\delta)}\right)\frac{1}{1-\delta\pi_A}(\pi_B)^2 \leq 1$$

Note that since $\frac{1+\delta(1+\pi_\ell)}{(1+2\delta)} < 1$, it is enough to show that

$$\frac{1}{1-\delta\pi_B}(\pi_A)^2 + \frac{1}{1-\delta\pi_A}(\pi_B)^2 \leq 1$$

But $1-\delta\pi_B > \pi_A$ and $1-\delta\pi_A > \pi_B$, so

$$\frac{1}{1-\delta\pi_B}(\pi_A)^2 + \frac{1}{1-\delta\pi_A}(\pi_B)^2 < \frac{1}{\pi_A}(\pi_A)^2 + \frac{1}{\pi_B}(\pi_B)^2 = \pi_A + \pi_B = 1,$$

which establishes the result.

7.4. **Contingent Payments.** In the paper we assumed that leaders offer instantaneous cash transfers in exchange for a commitment of support. Transfers that occurred in the past are sunk, and hence do not affect the incentives in subsequent periods. Alternatively one can assume that the leader and the follower agree on a contingent transfer in exchange for support; a "partnership" offer instead of a buyout. This in fact seems the most appropriate assumption in some applications, as in the case of endorsements by party elders in presidential primaries. In this case candidates negotiate with party elders their support, but they do so in exchange of future promises.

Substituting cash for promises can change the conditions for existence of a fully competitive equilibrium, but does not alter the conclusions regarding the ranking of competition and monopoly. Let $\tilde{W}_{out}(\vec{m}'|p_j(\vec{m}))$ denote the value in state $\vec{m}' = (m'_A, m'_B)$ of a committed follower locked with a promise $p_j(\vec{m})$ acquired towards leader j in state $\vec{m} = (m_A, m_B)$. Note that

$$\begin{aligned}\tilde{W}_{out}(\vec{m}'|p_A(\vec{m})) &= \sum_t \Pr\left(j \text{ wins in } t \text{ periods} \,|\, \vec{m}'\right)\delta^t[w_j + p_j(\vec{m})] \\ &\quad + \sum_t \Pr\left(\ell \text{ wins in } t \text{ periods} \,|\, \vec{m}'\right)\delta^t w_\ell \\ &= W_{out}(\vec{m}') + \underbrace{\sum_t \Pr\left(j \text{ wins in } t \text{ periods} \,|\, \vec{m}'\right)\delta^t p_j(\vec{m})}_{\tilde{p}_j(\vec{m}',\vec{m})},\end{aligned} \tag{65}$$

where $W_{out}(\vec{m}')$ denotes the value of a committed follower in state $\vec{m}'$ in the cash game, and $\tilde{p}_j(\vec{m}',\vec{m})$ gives the expected value of the contingent transfer $p_j(\vec{m})$ in state $\vec{m}'$. Note then that the value function $\tilde{W}_{out}(\vec{m}'|p_\ell(\vec{m}))$ is separable in transfers and the value derived from implementing the alternative. Thus when ℓ meets an uncommitted follower in state $\vec{m}$, she offers a contingent payment $p_\ell(\vec{m})$ such that

$$\tilde{p}_\ell(\vec{m}^\ell, \vec{m}) + \delta W_{out}(\vec{m}^\ell) = \delta \tilde{W}(\vec{m}) \tag{66}$$

This implies that the continuation payoff of a follower after he meets one of the leaders is $\delta\tilde{W}(\vec{m})$ no matter what, and therefore the recursive representation of $\tilde{W}(\vec{m})$ is given by (15) as in the "cash" game, so that $\tilde{W}(q,q) = W(q,q)$; i.e., the value of the uncommitted follower at the beginning of the promises game is equal to the value in the cash game. This moreover implies by (66) that the expected value of the payment in the promise game is the same as in the cash game.

Now, to evaluate existence of a fully competitive equilibrium (for our large n results), we need to consider the value of the leader. And in this regard there is in fact a crucial difference with the benchmark cash model. Note that since promises are executed if and only when the leader wins, present exchanges now affect the incentives for future exchanges and must be incorporated on the value function. In particular, the relevant state in the promises game is composed of the number of additional followers that each leader needs in order to win, as before, but now also the stock of promises that a leader brings to the table when meeting another follower.

This difference complicates the algebra, but does not alter our main results. To see this note that after a leader wins, she obtains a payoff composed of a direct benefit $\overline{v}_\ell$ and a transfer from/to all committed followers. These two components are, indeed, additively separable. Moreover, this property still holds recursively, which implies that the value function of the leader in any state – now with the stock of promises as part of the state – is also additively separable in the utility for winning and the promises collected if and when she wins. It follows immediately that Proposition 4.1 extends to this case and a fully competitive equilibrium exists for sufficiently high $\overline{v}$ or $\overline{v} - \underline{v}$. A similar argument holds for the monopoly case, and the welfare comparison in the paper holds.

7.5. **Simultaneous Contracting in the Monopolistic Model.** Here we compare simultaneous and sequential contracting in the public goods model with a single alternative. To consider the static setting, we follow Segal and Whinston (2000). Segal and Whinston consider two contracting environments: one in which the leader can make discriminatory offers (allowing $p_i \neq p_j$ for followers i, j), and one in which the leader cannot discriminate among followers, so that she has to offer a single offer p. We consider each in turn.

As a benchmark, recall that in the sequential monopoly game, the leader's equilibrium payoff is given by

$$v\left(m\right)=\delta^{m}v+\left[\sum_{l=1}^{m}\left(1-\delta\prod_{k=1}^{l}r(k)\right)\right]\delta^{m}w \tag{67}$$

As we explained in the paper, in the limit as $\delta \to 1$ followers' bargaining power is maximized, and the leader has no ability to extract rents, so that $v(m) \to v$. For $\delta < 1$, however, followers' bargaining power decreases, and the leader can in fact extract rents from the followers. The effect of reducing the discount factor on the leader's value at the beginning of the game then exhibits a tradeoff between a higher ability to extract rents from followers and a larger loss from discounting. When w is small, the effect of discounting dominates, and the leader's value increases monotonically with δ approaching v as $\delta \to 1$. When w is larger, the rent extraction effect dominates for high δ, and the leader's equilibrium payoff is maximized for a $\tilde{\delta} \in (0, 1)$. For low enough δ, of course, the discounting effect dominates, and the leader's value falls monotonically with δ, with $v(m) \to 0$ as $\delta \to 0$. In this case the simultaneous game obviously gives the leader a larger payoff than the sequential offer game.

Consider next the static game with *nondiscriminatory offers*. First, note that there exists an equilibrium in which all followers accept offers $p \geq 0$ and reject offers $p < 0$, and the leader offers $p = 0$. For any $p \geq 0$ all followers accept, so a follower i cannot gain by deviating to reject (would only lose p). For $p < 0$, all followers reject in equilibrium. If i were to deviate and accept, she would lose p. Given followers' strategy, the leader cannot gain by offering $p' \neq 0$: any $p' > 0$ would only mean larger transfers to the followers, and $p' < 0$ would lead her to lose. Note that neither leader nor followers are using weakly dominated strategies. In this equilibrium, the leader obtains a payoff of v, and does not extract rents from the followers.

There also exists an equilibrium in which the leader obtains a payoff $v + \frac{n+1}{2}w$. Suppose that for any $p \in [-w, 0)$ followers $1, 2, \ldots, (n+1)/2$ accept, and $(n+3)/2, \ldots, n$ reject, for $p \geq 0$ all followers accept, and for all $p < -w$ all followers reject. The leader offers $p = -w$. Consider first $p \in [-w, 0)$. Followers $i \geq (n+3)/2$ don't have an incentive to deviate, for the leader is already winning, and would only do worst by accepting the offer. Followers $i \leq (n+1)/2$ are pivotal, and thus as long as $p \geq -w$, have no profitable deviations. As before, for $p \geq 0$ or $p < -w$, followers have no profitable deviations. The leader doesn't have a profitable deviation. In equilibrium, she obtains a payoff $v + \frac{n+1}{2}w$. If she were to offer $p < -w$, she would get $0 < v + \frac{n+1}{2}w$. If she were to offer $p \in (-w, 0)$ she would only be reducing rent extraction. If she were to offer $p \geq 0$ she would get $v < v + \frac{n+1}{2}w$.

Next consider the static game with discriminatory offers. As in the previous case, with discriminatory offers there is a SPE in which the leader can extract all the surplus from $(n+1)/2$ followers. In particular, suppose that the leader offers $p = -w$ to a set I of $(n+1)/2$ followers, and $\tilde{p} < -w$ to a set O of $(n-1)/2$ followers, and that followers accept any offer $p_i \geq -w$ and reject any offer $p_i < -w$. Followers in O do not have a profitable deviation, since accepting gives them a payoff $w - \tilde{p} < 0$. Followers $i \in I$ get an equilibrium payoff of zero. But since each $i \in I$ is pivotal in equilibrium, rejecting doesn't improve his payoff. The leader cannot gain by proposing a different $\vec{p}$: whenever more than $(n+1)/2$ followers accept offers $p_i < 0$ for sure, any one follower would prefer to deviate and reject the offer. And increasing offers in I can only hurt the leader since it would decrease rents and not improve the probability of winning. The leader wins with probability one, and makes a payoff $v + w(n+1)/2$.

Is there an equilibrium without WDS in which the leader only makes v? Suppose all followers accept any offer $p_i \geq 0$ and reject any offer $p_i < 0$. Suppose the leader offers 0 to a set I of $(n+1)/2$ followers and $p \in (-w, 0)$ to a set O of $(n-1)/2$ followers. Consider a follower $i \in O$. In equilibrium, he gets w. By deviating and accepting the offer, he gets $w + p_i < w$. The followers in I don't have incentives to deviate either. In equilibrium the leader wins with probability one, and then gets a payoff v. Given the strategy of the followers, she cannot profitably deviate. This strategy profile thus is a SPE. Moreover, note that no player uses a weakly dominated strategy.

Both with discriminatory and non-discriminatory offers, there is an equilibrium in which the leader makes a payoff v and an equilibrium in which the leader makes a payoff $v + w(n+1)/2$. Note that the parenthesis in (67) is $\left(1 - \delta \prod_{k=1}^{l} r(k)\right) < 1$. Substituting,

$$v\left(q\right) < \delta^{q}\left[v + \left(\frac{n+1}{2}\right)w\right] < v + \left(\frac{n+1}{2}\right)w$$

Since $v < v(q)$ for w, δ large, it follows that in this case there is an equilibrium in the static game in which the leader obtains a payoff that is larger than her payoff in the unique equilibrium with sequential offers, but also equilibria in which the opposite holds.

Conclusion. For low δ and/or sufficiently small w, the leader's payoff with simultaneous offers is higher than in the sequential game. For w, δ large, instead, both with discriminatory and non-discriminatory offers there is an equilibrium in which the leader would prefer simultaneous to sequential offers, but also equilibria in which the opposite holds.