BEYOND DELEGATION: AGENCY DISCRETION WHEN BUDGETS MATTER

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Abstract. A fundamental result in the literature on congressional control of the bureaucracy is that optimal institutions take the form of limited delegation of decision-making authority to the agency. We revisit the question of optimal institutional design for cases in which the extent to which the policy is funded determines its effectiveness. We show that in this context, delegation is not optimal. Instead policy is almost everywhere below the agent's preferred policy. The optimal separating mechanism entails larger reductions of the budget for policies that go further in the direction of the agent's preferences; even more, in some states, than what the legislator herself would want to implement given full information.

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1. Introduction

A key question in political science is how elected representatives control bureaucrats. The basic issue at the heart of the problem is that while elected officials have final authority over decisions, bureaucrats are generally better informed about factors that determine the costs and benefits of alternative policies. To deal with this informational asymmetry, politicians design institutions that allows them to increase their control over the bureaucracy. How far can politicians limit the power of these unelected agents, while still incorporating their information in policy outcomes?

The early theoretical literature addressing this problem focused predominantly on agencies' budgets. In this context, the agency exchanges a lump-sum budget for a promised amount of "output" (Niskanen (1971), Miller and Moe (1983), Bendor, Taylor, and Van Gaalen (1985, 1987), Banks (1989)).[1] Following the work of Holmström (1977), however, theoretical work in political science shifted away from budgetary issues, and focused almost exclusively on informational asymmetries in an "ideological" dimension.[2] In this context, the agent has private information about the realization of a shock that affects the preferred policy of both the agent and the legislature, and legislators choose institutions to shape their interaction with the agency.

The literature's choice to focus on a single dimension of conflict was not capricious. The problem of institutional design with multiple, possibly interrelated dimensions,

[1]In Niskanen (1971), the agency is privately informed about the costs and benefits of its services relative to the legislature. Niskanen assumes that the agency is in complete control of budgetary agenda, reducing the analysis to that of a single actor. In Miller and Moe (1983) and Bendor, Taylor, and Van Gaalen (1985), the agency announces a supply schedule, from which the legislature selects the quantity of service to be produced. Bendor, Taylor, and Van Gaalen (1987) develop a model with legislative auditing, where the legislature selects a schedule that describes the agency's budget as a function of the agency's announced service cost. Banks (1989) considers a model with legislative auditing with no commitment.

[2]Bendor and Meirowitz (2004) and Gailmard and Patty (2012) provide an overview of this literature, and a general framework for this family of models. Two notable exceptions are Ting (2001) and McCarty (2004).

is difficult. This is particularly the case when the "principal" (e.g., Congress) cannot use transfers to alleviate incentive problems, as it often is the case in politics. However, when the principal contracts with the agent over a unidimensional space, the solution to the principal's problem is elegant and, ultimately, simple. In this situation, a principal who can choose among a large space of contracts will simply *delegate* decision-making authority to the agent over a set of possible actions. The principal's problem is then reduced to determining how much discretion to delegate to the agent.[3,4]

The substantial literature exploring the delegation solution in political science made significant progress to understand Congressional control of the bureaucracy, particularly in realms of policy that do not require funding. In many instances, however, the effect of a policy is inevitably linked with its scale of implementation. As Calvert, McCubbins, and Weingast (1989) have noted, most policies have a budgetary component, and "it is reasonable to suppose that the extent to which the agency is funded will determine the effectiveness of the agency's policy".[5]

The immediate counterpart of this logic is that "control over agency budgets is a critical tool of political influence in regulatory decision making" (Carpenter (1996); see also Weingast and Moran (1983), Bendor and Moe (1985), Cooper and West (1988), Wood (1990), Wood and Anderson (1993)). The key point we want to stress

[3]See Holmström (1977), Melumad and Shibano (1991), Alonso and Matouschek (2008). See Ambrus and Egorov (2012) and Amador and Bagwell (2012, 2013) for a variant of this problem with money burning.

[4]The optimal delegation literature presupposes that the principal can commit to a *mechanism* (or set of institutions) regulating her interaction with the agent. At the opposite extreme of the spectrum, Crawford and Sobel (1982) assume that the principal cannot commit to a policy choice. In this context, the principal will always choose her preferred policy given the information provided by the agent, and as a result cannot reward the agent with policy concessions after the agent reveals her information.

[5]This was examined in detail by Weingast and Moran (1983) in the case of the FTC, by Wood (1990) in the case of the the Equal Employment Opportunity Commission (EEOC), and by Wood and Anderson (1993) in the case of the Antitrust Division of the Justice Department.

here is that budget control allows the political principal to affect agencies' *policy* decisions.[6] In this context, the legislator can, and often does, use the budget assigned to a particular policy to provide incentives to the agency, by distorting both the scale and content of policy outcomes.

In this paper, we characterize the optimal institutional arrangement for the legislature in this setting. In our model, the agent has private information about the realization of a shock that affects the preferred policy of both the agent and the legislature, as is common in the literature. However, we assume that policy outcomes are composed of a policy, and a policy budget or scale of implementation, which are both relevant for the agency and the legislator. In particular, we assume that the marginal value of the budget assigned to the policy is decreasing in the distance between the policy and the ideal policy of each actor. To fix ideas, consider the following simple examples:

Bailouts to Small and Large Firms. Consider bailouts to large and small firms in a financial crisis. In this context, we think of the policy as how much of the rescue effort should be devoted to large firms, relative to small firms, and of the policy budget as the total amount of the bailout. We assume that the Treasury Department is better informed than Congress about the effectiveness of rescuing small and large firms, but has a bias towards large firms. Based on its information, Treasury reports to Congress a proposed allocation of money to small and large firms. The policy outcome is a bill stipulating a total bailout amount, and how the money should be allocated to small and large firms.

NSF Research Funding. Think of the policy is the proportion of the budget going to a new field, like Data Science, and the policy budget as the total NSF research

[6]As Pasachoff (2015) points out in her detailed description of the Resource Management Offices within the Office of Management and Budget (OMB), "their authority over budget preparation, budget execution, and related management initiatives gives them a wide purchase over agency policy decisions."

budget. In this context, NSF is better informed than the legislator about the optimal proportion of money to be allocated to Data Science relative to traditional fields, and has a bias towards Data Science relative to the legislator. Based on its information, NSF budget request includes a recommendation to allocate a set proportion of the budget to Data Science. The policy outcome is a proportion of money to be allocated to Data Science, and the total research budget for NSF.

Homeland Security. In this context, we can think of the policy as the proportion of total resources that should be allocated to Border Patrol relative to Cyber Security. DHS is better informed than the legislator about the threat of cyber attacks relative to illegal immigration. For any realization of the state, DHS has a bias towards cyber relative to the legislator. Based on its information, DHS's budget request stipulates a proposed allocation of funds, with a justification based on their assessment of the relative importance of cyber vs. immigration. The policy outcome is an allocation of the DHS budget to Border Patrol and Cyber, and a total DHS budget.

To allow a rich contract space, we take a mechanism design approach, in the spirit of the delegation solution. In Section 3 we characterize the solution when the state is binary. In Section 4 we extend our analysis to the continuum.

We show that when the scale of implementation matters, delegation is not optimal.[7] In our context, instead, the optimal incentive compatible policy is below the agent's preferred policy almost everywhere. In our bailout example, for instance, this would mean that large firms receive a smaller allocation than what Treasury would prefer for all realizations of the state. Furthermore, the optimal separating mechanism entails larger reductions of the budget for policies that go further in the direction of the agent's preferences; even more, in some states, than what the legislator herself would

[7]In the delegation literature, giving the agent complete discretion over a range of policies emerges as an optimal mechanism. This is because in any optimal mechanism, the policy function coincides with the agent's ideal policy in this range.

want. In the example, this means that when it is in fact optimal to focus on large firms, the size of the bailout is smaller than what both Treasury and Congress would want.

The model makes clear that equilibrium budget and policy are linked, because there is a tradeoff between policy responsiveness to information and distortions in the scale of implementation.[8] In states in which the legislator would want to focus predominantly on small firms, the equilibrium allocation of funds to small and large firms is a compromise between the preferred allocations of Treasury and Congress, and the size of the bailout is larger than the first best for the legislator. For intermediate realizations of the state, the optimal incentive compatible policy still lies between Congress' and Treasury's preferred allocation, but the total size of the bailout is inefficiently low, below Congress' and Treasury's first best. Finally, in states in which Congress would want to heavily focus on larger firms, both the relative allocation of funds to large firms *and* the total size of the bailout are below the first best for both Congress and Treasury.

In the paper we analyze how the solution varies with the level of conflict of interests between agency and legislator. As conflict increases, policy becomes less responsive to the state. The range of variability of the budget, on the other hand, is non-monotonic in the level of conflict of interests. Increasing the agent's bias from a low initial level leads to lower variability of policy and a higher variability in budget levels, but a similar increase from a high initial level leads to lower overall variability in policy *and* budget levels.

While there has been an extensive literature on delegation, the solution to the class of problems we consider here has not been explored. Ting (2001) and McCarty (2004)

[8]For simplicity, we summarize these results in the context of our bailout example, focusing on the results for the continuum. Similar results obtain in the binary state model.

study models in which the agency's actions are costly, and the legislature chooses a budget for the agency. These papers, however, don't consider the optimal mechanism for the legislature.[9]

Baron (2000) and Krishna and Morgan (2008) analyze the unidimensional policy space with transfers, assuming quasilinear preferences and quadratic policy payoffs. In this context, transfers can provide incentives, but by definition do not change the effectiveness of the policy. The multidimensional case without transfers is less common in the literature; Koessler and Martimort (2012) study a two-dimensional policy space with separable quadratic payoff. In our case, instead, the content and scope of policy are complements in the utility function of principal and agent. Thus, we have what Koessler and Martimort call "externalities across decisions".

The results from our model are fundamentally different from both the delegation solution, and from the models with transfers. In the former, whenever policy is responsive to the state, it has to coincide with the agent's preferred policy (thus delegation is optimal). In the latter, whenever policy is responsive, it lies between the preferred points of agent and legislator, and tilts towards the agents' preferred policy in higher states, possibly coinciding with the agent's preferred policy in an interval.

2. The Model

A legislator chooses institutions to regulate the behavior of an agent, who has private information about a payoff-relevant state variable $\omega \in \Omega$. It is common knowledge

[9]In Ting (2001), the agency can choose a more right winged policy at a cost, which enters its quasilinear utility function as a transfer. Congress initially chooses a budget for the agency and, after observing a signal of the agency's choice of policy, an auditing level. In McCarty (2004), the agency needs resources to move policy away from the status quo. The President appoints the agent, while Congress chooses the agency's budget, and thus effectively a range of discretion for the agency around the status quo.

that $\omega \sim F$, where we assume that $supp\,(F) = \Omega$. We consider two cases. In section 3, we analyze the binary state case, in which $\Omega = \{0, 1\}$. In section 4 we extend our analysis to the continuum case, in which $\Omega = [0, 1]$.[10]

An outcome $(x, m) \in \mathbb{R} \times \mathbb{R}_+$, comprises a policy x, and a policy budget, or scale of implementation m. The legislator and agent have state-contingent preferences, $V(x, m|\omega)$ and $U(x, m|\omega)$, respectively. The legislator has ideal policy ω, and prefers a limited budget $\hat{m}(\omega) > 0$. Thus, the marginal value of increasing project size is negative when $m > \hat{m}(\omega)$, and positive when $m < \hat{m}(\omega)$, but in either case increasing in absolute value as policy x is closer to her ideal policy ω; i.e., the legislator cares more about setting project size to the ideal goal when policy is closer to its preferred policy. In particular, we assume that the principal's payoff is given by $V(x, m|\omega) = v((\omega - x)^2, (m - \hat{m}(\omega))^2) > 0$, where v is twice differentiable, decreasing in both arguments and $v_{11} \leq 0, v_{22} \leq 0$, $v_{12} \leq 0$. We assume V is concave in x, m. Given our previous assumptions a sufficient condition for this is $(v_{12})^2 \leq v_{11}v_{22}$.

We assume that the agent always prefers a larger scale of implementation, and has ideal policy $\omega + b$, where $b > 0$. In particular, we assume that $U(x, m|\omega) = u\left((\omega - x + b)^2, m\right) > 0$, where u is decreasing in the first argument and increasing in the second. We also assume that U is quasiconcave (therefore has convex better-than sets) and satsifies single-crossing, i.e., for fixed x, m and any $\omega > \omega'$ we have

$$\frac{U_x(x, m|\omega)}{U_m(x, m|\omega)} > \frac{U_x(x, m|\omega')}{U_m(x, m|\omega')}.$$

We consider the problem of maximizing the legislator's payoff by choosing optimal state-contingent institutions to regulate the behavior of the agent. To allow a rich contract space, we take a mechanism design approach. Without loss of generality,

[10] In our running example, we could interpret x as the proportion of bailout funds going to big firms. By scaling preferences appropriately (see footnote 15), we can ensure this stays between zero and one.

we consider direct truthful mechanisms, in which the legislator proposes a menu of contracts $\{(x(\omega), m(\omega))\}_{\omega\in\Omega}$ to the agent, and is committed to implementing the policy $(x(\hat{\omega}), m(\hat{\omega}))$ if the agent announces that the realized state is $\hat{\omega} \in \Omega$.

By the revelation principle (Myerson (1979), Dasgupta, Hammond, and Maskin (1979)), the outcomes of any optimal mechanism can be implemented by a truthful direct mechanism. Thus, while we will not recover the particular protocol that the legislator might be using, the solution will capture the equilibrium relation between states and outcomes. Throughout, we will restrict to deterministic mechanisms. This assumption seems eminently plausible in the application to legislative control of the bureaucracy.

3. Binary State Space

We begin by studying the model with a binary state space, $\Omega = \{0, 1\}$. This allows us to present the analysis in a simpler environment, and to examine in more detail how the parameters of the problem shape the solution. In this context, the legislator's problem is to choose $(x(0), m(0))$ and $(x(1), m(1))$ to maximize

$$\sum_{\omega\in\{0,1\}} f(\omega)V(x(\omega), m(\omega)|\omega)$$

subject to the incentive compatibility (IC) constraints:

$$U(x(\omega), m(\omega)|\omega) - U(x(\omega'), m(\omega')|\omega) \geq 0 \quad \text{for } \omega, \omega' \in \{0, 1\}.$$

Since the legislator's preferences depend on the realization of the state, the first best for the legislator entails different policies and (possibly) a different scale of implementation of these policies in each state. Because of the conflict of interests between the legislator and the agent, though, achieving this result can require distortions of the budget and policy plans. If these distortions are too costly for the legislator, the legislator would be better off by choosing a policy and budget plan that is not contingent on the realization of the state. Our first result shows that – in the binary state environment – *generically* this doesn't happen. Instead, it is optimal for the legislator to give the agent some discretion over policy outcomes.[11]

Proposition 3.1 (No Pooling)**.** *A pooling contract, where* $x(0) = x(1) = x_p$ *and* $m(0) = m(1) = m_p$*, is generically suboptimal for the legislator.*

The key implication of proposition 3.1 is that policy outcomes will be *responsive* to the private information of the agent. In the context of our bailout example, the equilibrium allocation of funds between small and large firms and the size of the bailout are generically responsive to the recommendations by the Treasury Department.

The nature of the solution depends on the level of conflict of interests between the legislator and the agent. First, if the conflict of interest is sufficiently low – for the case where $\widehat{m}(1) = \widehat{m}(0) = \widehat{m}$ this means $b \leq 1/2$ – incentive constraints will not be binding in the solution, and the legislator will be able to achieve her first-best policy in each state.[12] For a larger conflict of interests, however, achieving incentive compatibility will necessarily imply policy distortions relative to the legislator's first best, which are themselves costly to the legislator. To induce Treasury to report its information truthfully, the allocation and size of the bailout induced by different reports must be different than what Congress would prefer to do knowing that information.

Even in the binary state, characterizing the solution to the legislator's problem is somewhat complicated, because it depends on both the legislator's and committee

[11] We consider the topological notion of genericity – where a property is generic if it is satisfied in an open dense set, and not generic if it is satisfied only in a closed nowhere dense set. We show that the utility functions which satisfy a necessary condition for pooling are nowhere dense in the appropriate Sobolev space, $W^{1,p}(X)$.

[12] More generally, the principal can implement the first-best if $u\left(b^2, \hat{m}(0)\right) \geq u\left((b-1)^2, \hat{m}(1)\right)$.

preferences, as well as the probability distribution over the states $f(\cdot)$. However, the problem simplifies if we break the problem in two parts: first we establish "contract curves" on which a solution must lie using only the payoff functions; second, the probability distribution $f(\cdot)$ determines which point on the contract curves should be chosen.

Given that the binding incentive constraint is that of state 0, attaining incentive compatibility for the agent entails making policy in state $\omega = 1$ *less* attractive to the agent and/or policy in state $\omega = 0$ *more* attractive to the agent in the least costly manner for the legislator. In our running example, this means that relative to Congress' first best, the allocation and size of the bailout have to be more attractive to Treasury when it seeks to favor small firms, and less attractive to Treasury when it seeks to favor large firms.

Our assumptions imply that first-order conditions can be used to characterize the solution of the optimization problem.[13] From the FOCs of the legislator's problem, in the optimal contract

$$\frac{V_x(x(\omega), m(\omega)|\omega)}{V_m(x(\omega), m(\omega)|\omega)} = \frac{U_x(x(\omega), m(\omega)|0)}{U_m(x(\omega), m(\omega)|0)} \quad \text{for } \omega = 0, 1. \tag{1}$$

where the agent's marginal rate of substitution is *evaluated at his state 0 preferences*, for both realizations of the state.

For each ω, equation (1) characterizes a state-contingent contract curve; a *reward curve* in state 0, $CC(0)$, and a *discipline curve* in state 1, $CC(1)$. The reward curve, $CC(0)$, is the set of points $\{\tilde{x}^0(u^*), \tilde{m}^0(u^*)\}$ which maximize the principal's utility in state 0, subject to the agent obtaining at least utility u^* in state 0; that is

$$(\tilde{x}^0(u^*), \tilde{m}^0(u^*)) := \arg\max_{(x,m)} V(x, m|0) \text{ s.t. } U(x, m|0) \geq u^*.$$

[13]See Theorem 3 in Arrow and Enthoven (1961).

Conversely, the discipline curve, $CC\,(1)$, is the set of points $\{\tilde{x}^1(u^*), \tilde{m}^1(u^*)\}$ which maximize the principal's utility in state 1, subject to giving the agent at most utility u^* in state 0; that is

$$(\tilde{x}^1(u^*), \tilde{m}^1(u^*)) := \arg\max_{(x,m)} V(x, m|1) \text{ s.t. } U(x, m|0) \leq u^*.$$

Figure 1 plots these two curves. Each value of u^*, corresponds to one point on $CC\,(0)$ and one point on $CC\,(1)$. The relevant values of u^* for the principal's problem belong to $W = [U(\hat{x}_0, \hat{m}_0|0), U(\hat{x}_1, \hat{m}_1|0)]$, where $(\hat{x}_\omega, \hat{m}_\omega)$ denotes the legislator's first-best policy in state ω. The reward and discipline curves maximize the legislator's profit while making the policy in state $\omega = 1$ less attractive to the agent (than u^*) and the policy in state $\omega = 0$ more attractive to the agent (than u^*).

Because policy/budget combinations on the contract curves reward the agent in state 0 and discipline the agent in state 1 efficiently, a policy/budget plan lying anywhere outside the contract curves can be improved with an alternative plan that preserves incentives and increases the legislator's utility. Note that from the single crossing property, along the discipline curve

$$\frac{V_x(x(1), m(1)|1)}{V_m(x(1), m(1)|1)} = \frac{U_x(x(1), m(1)|0)}{U_m(x(1), m(1)|0)} < \frac{U_x(x(1), m(1)|1)}{U_m(x(1), m(1)|1)}.$$

This means that in state 1, the agent has a higher willingness to pay for policy relative to implementation scale than the legislator, and both the legislator and the agent could gain by a trade that increases policy and reduces implementation scale. In the reward curve, instead, the optimal plan equalizes the agent's and legislator's marginal rate of substitution of budget for policy in state 1.

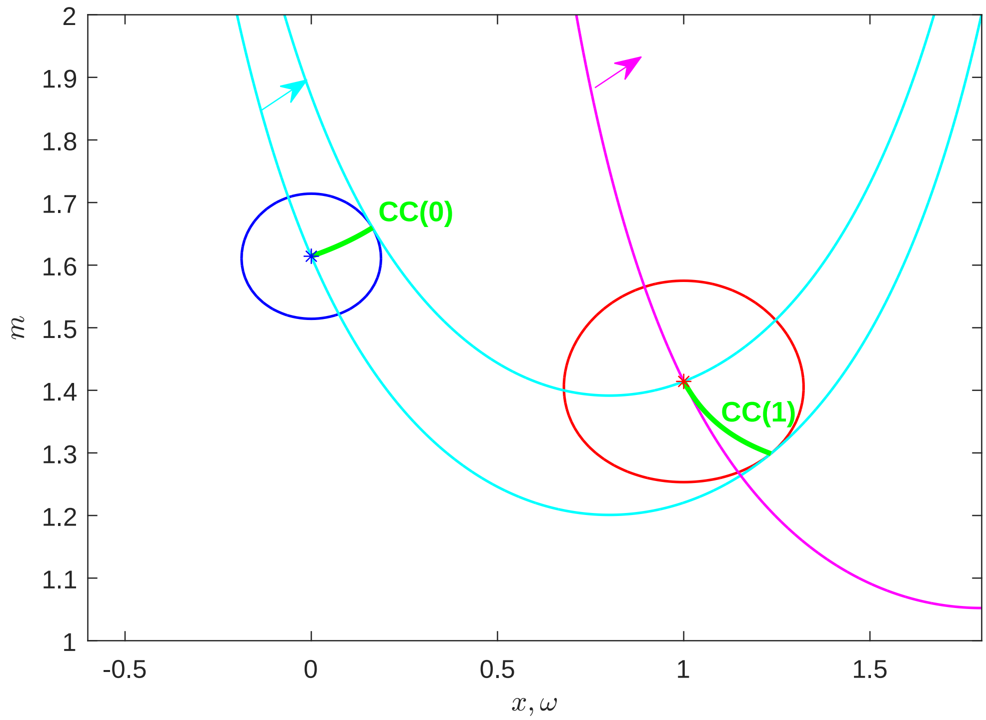


FIGURE 1. Contract Curves, $CC\,(0)$ and $CC\,(1)$. The principal's indifference curves are the dark blue (state 0) and red (state 1) ovals. The agent's utility in state 0 is the light blue line, with the arrow showing the direction of increasing utility. The reward curve are the tangency points between the dark blue and light blue indifference curves, while the discipline curve is given by the tangency points between the red and light blue indifference curves.

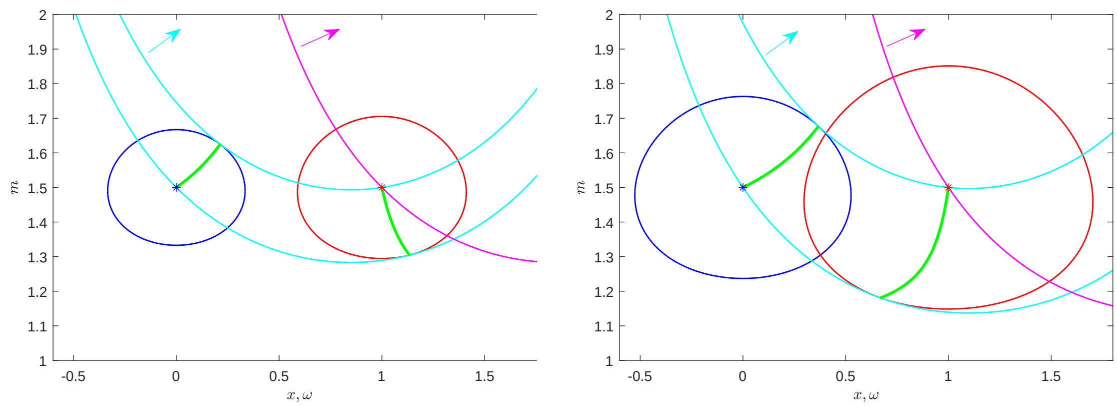
Since the legislator is choosing between pairs of points on the contract curves, we can now rewrite the legislator's problem as:

$$\max_{u \in W} \; f(0)V(\tilde{x}^0(u), \tilde{m}^0(u)|0) + f(1)V(\tilde{x}^1(u), \tilde{m}^1(u)|1), \tag{2}$$

where $(\tilde{x}^\omega(u), \tilde{m}^\omega(u)) \in CC\,(\omega)$ are points on the relevant contract curves.

To further illustrate the nature of the optimal policies, we now consider the case where $\hat{m}(0) = \hat{m}(1)$, i.e., the principal wants the same scale of implementation in both states.

The contract curves allow us to characterize the nature of the distortions in policy and scale of implementation. To do this it is useful to distinguish two cases. We say that the conflict of interest between the legislator and the agent is *moderate* if the agent's ideal policy in state 0 is below the first best policy for the legislator in state 1; i.e., if $b < 1$. We say that the conflict of interest between the legislator and the agent is *large* if $b > 1$. Note that since in the continuum local deviations are "small" relative to the size of the bias, we think of the large conflict of interests case as the natural benchmark for comparison with the continuum case which we analyze in Section 4.



Moderate Conflict ($b \in (1/2, 1)$) Large Conflict ($b > 1$)

FIGURE 2. Moderate and Large Conflict of Interests.

The left panel of Figure 2 plots representative contract curves for moderate conflict of interests. As the figure illustrates, the reward curve is increasing, and the discipline curve is decreasing. This means that in the second-best plan, policy will be larger than the first best for the legislator in both states, i.e., $x(\omega) > \omega$ for $\omega = 0, 1$. On the other hand, the scale of implementation will be *larger* than the first-best for the legislator in state $\omega = 0$, but *smaller* than the first-best for the legislator in state $\omega = 1$. In our running example, this says that in the solution for moderate conflict of interests between Congress and Treasury, the relative allocation of funds to large firms is larger than what Congress would want to implement in both states. Moreover, the total size of the bailout is larger than the first best for Congress when Treasury recommends favoring small firms, but lower than the first best for Congress when Treasury recommends favoring large firms.

The intuition for the result can be seen graphically in Figure 2. When $b < 1$, the state 1 indifference curves for the legislator and the state 0 indifference curves for the agent are tangent below and to the right of the legislator's ideal point in state 1. Because the ideal policy of the agent in state 0 is still lower than the ideal policy of the legislator in state 1, the least costly way to leave the agent at some utility level u below than what he would obtain at $(\hat{x}(1), \hat{m}(1))$ is to reduce the scale of the program m, and increase the policy x.

When conflict of interests between the agency and the legislator is large, instead, the state 1 indifference curves for the legislator and the state 0 indifference curves for the agent are tangent below and to the *left* of the legislator's ideal point in state 1. Therefore both the reward curve $CC(0)$ and the discipline curve $CC(1)$ are increasing, as in the right panel of Figure 2. This means that while the implementation scale of the policy in state $\omega = 1$ will be smaller than in the first-best as in the previous case, the direction of the policy outcome in state $\omega = 1$ will now be distorted *against* the direction of the agent's bias. In our example, this says that the allocation of funds to large firms when large firms should receive aid is smaller than what Congress would want to implement in the first best.

The reasoning is symmetric to the previous case: when $b > 1$, the ideal policy of a state 0 agent is larger than the ideal policy of the legislator in state 1. Thus, the least costly way to leave the agent at some utility level u below what he would obtain at $(\hat{x}_1, \hat{m}_1)$ is now to *decrease* the policy x and reduce the implementation scale m.

The next proposition summarizes the previous discussion.[14]

Proposition 3.2. *Let $\Omega = \{0,1\}$, and suppose $\hat{m}(0) = \hat{m}(1) = \hat{m}$. When $b > 1/2$, the optimal incentive compatible solution for the legislator has the following properties:*

(1) *The scale of implementation is larger (smaller) than the first best level for the legislator in state 0 (in state 1); i.e., $m(0) > \hat{m}$ and $m(1) < \hat{m}$.*
(2) *For moderate conflict of interests, policy is higher than the first best for the legislator in both states; i.e., $x(\omega) > \omega$ for all $\omega \in \{0,1\}$. For large conflict of interests, instead, $x(0) > 0$ and $x(1) < 1$.*

Proposition 3.2 characterizes the qualitative nature of the distortions and was entirely independent of the legislator's prior, f.*Exactly how much* the legislator distorts policy in each state depends on the likelihood of each state, which we can see from (2). In particular, the first order conditions at the optimal level u^* (interior by Proposition 3.2) imply that:

$$\frac{f(0)}{1-f(0)} = -\frac{\partial V(\tilde{x}^1(u), \tilde{m}^1(u)|1)/\partial u}{\partial V(\tilde{x}^0(u), \tilde{m}^0(u)|0)/\partial u}.$$

Note that the optimal trade-off between distortions in state $\omega = 1$ and state $\omega = 0$ depends on the likelihood of each state. As the probability of the low state $\omega = 0$ increases, we "move down the contract curves" in Figure 2, reducing the size of the distortion in state $\omega = 0$ in exchange for an increased distortion of the policy and budget plan in state $\omega = 1$. We state these results in the following remark:

Remark 3.3. *As state $\omega = 0, 1$ becomes more likely, the distortions in the policy and budget plan in state ω are less severe, and the distortions in state $1-\omega$ are more severe. In particular, as $f(0)$ increases:*

(1) *the policy budget decreases in both states,*

[14] A similar, albeit more complicated characterization is possible if $\hat{m}_0 \neq \hat{m}_1$.

(2) *policy moves against the direction of the agent's bias in state 0, and*

(3) *when conflict of interests are moderate (large), policy moves in (against) the direction of the agent's bias in state 1.*

Consider the implications of Remark 3.3 in our running example. Suppose we observe policy outcomes in two cases, A and B, that only differ in that $f^B(0) > f^A(0)$ (the "small firm" state is more likely in case B). Then we would expect that the total size of the bailout would be lower in case B relative to case A, following any report by Treasury. We would also expect that in the small firm state, small firms would receive a larger fraction of the bailout in case B relative to case A. The comparative static regarding the fund allocation in the "large firm" state depends on the conflict of interests. In particular, we would expect large firms to get a larger fraction of the bailout in case B for moderate conflict of interests, but a smaller fraction for large conflict of interests.

It is now also easy to see how the optimal plan responds to changes in the level of conflict of interests. Consider a point in the disciplining curve $CC(1)$ for an agent with bias $b' < 1$, and suppose that we increase the agent's bias from $b' < 1$ to $\tilde{b} \in (b', 1)$. Because in state 0 the $\tilde{b}$ bias agent's preferred policy is closer to the ideal policy of the legislator in state 1, this agent is willing to make a larger policy concession for a given increase in budget (the agent has a flatter indifference curve through the point). Thus, the disciplining curve for an agent $\tilde{b} \in (b', 1)$ will be steeper than for b'. This means that the optimal incentive compatible plan for the agent with bias $\tilde{b}$ will entail a sharper reduction in budget and a smaller change in policy in state one relative to state 0. In the extreme, for $b = 1$, the most efficient way to punish the (state-0) agent in state 1 is to reduce the implementation scale without changing policy.

As we continue to increase the bias of the agent above $b = 1$ (in the "large bias" case), increases in the value of the state 1 implementation scale become less valuable for the state 0 agent relative to gains in policy, and the most efficient way to punish the agent is through small reductions in budget and sharp distortions in policy.

4. Continuum of States

We now extend our analysis to the case in which there is a continuum of states. In our running example, this means that the optimal state contingent allocation of funds between large and small firms for Congress is a number $\omega \in [0, 1]$.[15] Our main goal is to establish whether the nature of the distortions in policy we obtained in the two-type model extend naturally to the case in which there are multiple states. With this goal in mind, we will focus on characterizing the optimal fully separating contract, which we assume to be differentiable. We then provide conditions under which the fully separating contract dominates any pooling contract in this context.

Because of the complexity of this problem, we will assume throughout this section that the preferences of legislator and agent are given by[16]

$$V(x, m|\omega) = \left[A - (m - \hat{m})^2 - \alpha(\omega - x)^2\right]^{1/2} \tag{3}$$

[15]We can re-scale the variables ω, x and b by some η to ensure that the principal's and agent's optimal choice ($\eta\omega$ and $\eta(\omega + b)$, respectively) are between zero and one. This would result in the same model, except with the α and β parameters scaled by η^2. Even without this rescaling we will see that the optimal solution has $x(\omega) \in [0, 1]$.

[16] Assuming a specific utility function is standard in the literature. Baron (2000) and Krishna and Morgan (2008) assume quadratic policy payoffs in a unidimensional policy space with separability of transfers (i.e., a quasilinear utility function). Melumad and Shibano (1991) assume quadratic payoffs in a unidimensional policy space with no transfers. In the same context, Alonso and Matouschek (2008) assume quadratic payoffs for the principal, and a single-peaked symmetric utility function for the agent. Koessler and Martimort (2012) assume that payoffs are quadratic in each dimension and separable across dimensions. We deviate from the quadratic payoffs assumption that is prevalent in the literature because of the non-separability of payoffs that is at the core of our problem.

and

$$\tilde{U}(x,m|\omega) = Q\left(m - \frac{\beta}{2}(\omega + b - x)^2\right). \tag{4}$$

Here $\beta > 0$ ($\alpha > 0$) denotes how much the agent (legislator) cares about policy relative to scale of implementation, $b > 0$ is the agents' policy "bias" relative to the legislator, and $Q(\cdot)$ is an increasing, concave function. Since we will focus on deterministic contracts, and the state is realized and observed by the agent before the agent makes a decision, any monotonic transformation of the agent's utility function will represent the same preferences. Thus, in solving for the optimal incentive compatible problem, we will further simplify the agent's utility function to $U(x,m|\omega) = m - (\beta/2)(\omega + b - x)^2$.

With these assumptions, we are able to characterize the optimal menu of contracts in sufficiently rich detail so as to compare the results with the two-type case. We do this in theorems 4.5, 4.6 and 4.7. We begin by characterizing efficient allocations. Then move on to incentive compatible plans, and the optimal separating contract. In section 4.4, we provide conditions for the optimal contract to be fully separating.

4.1. **Efficiency.** Efficient allocations are given by the solution to the following problem, for $\lambda > 0$:

$$\max_{x(\cdot),m(\cdot)} \int [U(x(\omega),m(\omega)|\omega) + \lambda V(x(\omega),m(\omega)|\omega)] f(\omega) d\omega,$$

The first order necessary conditions are

$$\frac{V_x(\cdot|\omega)}{V_m(\cdot|\omega)} = \frac{U_x(\cdot|\omega)}{U_m(\cdot|\omega)} \tag{5}$$

and

$$U_x(\cdot|\omega) = -\lambda V_x(\cdot|\omega). \tag{6}$$

Conditions (5) and (6) define an efficient plan $(x^\dagger(\omega), m^\dagger(\omega))$ for any state ω. Condition (5) implies that in an efficient plan, the marginal rate of substitution (MRS) of legislator and agent between policy and scale of implementation are equalized at all $\omega \in \Omega$. Condition (6), on the other hand, has the implication that the optimal policy lies in the contract set for legislator and agent, $x(\omega) \in (\omega, \omega + b)$ (Lemma A.1.1). Note also that substituting (6) in (5) implies that $sg(U_m) \neq sg(V_m)$, so that in an efficient plan, $m(\omega) > \hat{m}$ for all $\omega \in \Omega$.[17]

4.2. **Incentive Compatible Mechanisms.** Now consider the legislator's problem. The legislator offers the agent a menu of incentive compatible contracts $(x(\cdot), m(\cdot))$, where $x : [0,1] \to \mathbb{R}$ and $m : [0,1] \to \mathbb{R}_+$. Here $x(\omega)$ and $m(\omega)$ denote, respectively, the policy and scale of implementation in state $\omega \in \Omega$. Letting $\mathcal{U}(\hat{\omega}, \omega) := U(x(\hat{\omega}), m(\hat{\omega})|\omega)$, the legislator's problem is:

$$\max_{\{x(\omega), m(\omega)\}} \int_0^1 V(x(\omega), m(\omega)|\omega) f(\omega) d\omega \tag{PP}$$

subject to:

$$\mathcal{U}(\omega, \omega) \geq \mathcal{U}(\hat{\omega}, \omega) \text{ for all } \omega, \hat{\omega} \in [0,1].$$

We begin by characterizing incentive compatible plans. The next result is standard.

Lemma 4.1. *If $(x(\cdot), m(\cdot))$ is incentive compatible, $x(\cdot)$ is non-decreasing. Moreover, provided that policy is never larger than the preferred policy for the agent in an interval $I \subset \Omega$ (i.e., $x(\omega) \leq \omega + b$ for all $\omega \in I$), then $m(\cdot)$ is non-increasing in I.*

[17] If we further assume that the first-best project size for the legislator $\hat{m}$ is not too low ($\hat{m} > \beta b^2$), then for any ω, the policy in the efficient solution is a linear function $x^\dagger(\omega) = \omega + \Delta$ for some $\Delta > 0$ (Lemma A.1.2).

To assure that the contract is incentive compatible for the agent, policy must be nondecreasing in the state. In the bailout example, this implies that the allocation of funds to small firms never increases when the Treasury requests that a larger proportion of funds be allocated to large firms. In addition, provided that policy is never above the preferred policy for the agent, the scale of implementation is non-increasing in the state; i.e., the overall size of the bailout never increases when Treasury requests that a larger proportion of funds be allocated to large firms.

The weak monotonicity of Lemma 4.1 is still consistent with the first-best solution for the legislator. However, incentive compatibility has further implications on the budget associated with each policy in the menu, which do rule out the legislator's first-best plan. The necessary condition for no profitable local deviations at any point of differentiability ω is that:

$$m'(\omega) = -\underbrace{\frac{U_x(x(\omega), m(\omega)|\omega)}{U_m(x(\omega), m(\omega)|\omega)}}_{MRS^a_{xm}(\omega)} x'(\omega) = -\beta(\omega + b - x(\omega))x'(\omega). \tag{7}$$

Thus, incentive compatibility implies that at any point $\omega \in \Omega$ at which the contract is differentiable, the rate of change of the budget in the optimal mechanism must be proportional to the rate of change in the policy, by a factor given by the agent's marginal rate of substitution in that state. This directly implies, in particular, that if the agent cares about policy ($\beta > 0$), and has a conflict of interests with the legislator ($b > 0$), the legislator's unconstrained first-best plan is not a solution to the legislator's problem. In particular, note that if $x(\omega) = \omega$ for all $\omega \in [0, 1]$, as in the legislator's first best, then $U_x(x(\omega), m(\omega)|\omega) \neq 0$ for all $\omega \in [0, 1]$. Thus (7) implies that $m'(\omega) \neq 0$ for all $\omega \in [0, 1]$, which is inconsistent with setting $m(\omega) = \hat{m}$.[18]

[18]Condition (7) is also sufficient to assure no profitable local deviations if $x(\cdot)$ is nondecreasing. In fact, a standard argument shows that given the single-crossing condition, if $x(\cdot)$ is nondecreasing, (7) is necessary and sufficient to rule out both local and global deviations.

In the next proposition we solve the differential equation in (7) to write the budget at each state $\omega \in \Omega$ in an incentive compatible plan as a function of state ω and the policy $x(\omega)$. This result is important because it allows us to re-express the incentive compatibility constraints in the legislator's problem – which appeared as differential equations – as simple functions that can be plugged-in directly into the objective.

Proposition 4.2. *Given the policy function $x(\cdot)$, in an incentive compatible plan, the budget at each state ω can be written as:*

$$m(\omega) = \frac{\beta}{2}(x(\omega))^2 - \beta(\omega + b)x(\omega) + \beta \int_0^\omega x(r)dr + \kappa(m_0, x_0), \tag{8}$$

where

$$\kappa(m_0, x_0) = m_0 - \frac{\beta}{2}(x_0)^2 + \beta b x_0$$

Suppose, for example, that the legislator implements a linear policy rule $x(\omega) = x_0 + \pi\omega$, for $x_0 < b$. Then (8) becomes

$$m(\omega) = -\beta\pi(b - x_0)\omega - \beta\pi \left[\frac{1-\pi}{2}\right]\omega^2 + m_0$$

This is a strictly decreasing function for all $\omega \in \Omega$ (given $x_0 < b$), which is linear when $\pi = 1$, and strictly concave (convex) if $\pi < 1$ ($\pi > 1$). The absolute value of the slope, $\beta\pi\left[(b - x_0) + (1 - \pi)\omega\right]$, is increasing in the weight the agent gives to the policy direction relative to scale of implementation, β, and increases faster with ω the more responsive the policy is to the state.

4.3. **Optimal Mechanisms.** In this section, we characterize the optimal deterministic separating contract, which we assume to be continuous.[19] Using the results of

[19]Two things make this assumption palatable. Firstly, the problem itself is very well behaved (e.g., the objective function is concave) and while the Mangasarian sufficient conditions don't apply (since the constraint is linear in the control), it would be surprising to expect a discontinuous solution. Secondly, in solving the model numerically, we place no continuity restrictions on the possible solutions and indeed find continuous solutions for all parameter combinations.

Section 4.2, we can write the legislator's problem (PP) as:

$$\max_{\{x(\omega),m(\omega)\}} \int_0^1 V(x(\omega), m(\omega)|\omega) f(\omega) d\omega$$

subject to the incentive compatibility condition (8), and the constraints that $x'(\omega) > 0$ and $m(\omega) \geq 0$ for all $\omega \in \Omega$.[20] As is standard practice, we solve the relaxed problem without the additional constraints, and then check that these additional constraints are satisfied in the solution.

We begin by showing that in an optimal mechanism, the policy budget $m(\cdot)$ is non-increasing in the state, and in fact strictly decreasing whenever $x(\omega) < \omega + b$. The second statement follows immediately from (7), since $x'(\cdot) > 0$. The first statement follows immediately from lemma 4.1 after showing that $x(\omega) \leq \omega + b$ for all $\omega \in \Omega$. The condition that $x(\omega) \leq \omega + b$ seems intuitive: if it were the case that $x(\omega) > \omega + b$ for some ω, the legislator could choose a policy $\tilde{x}(\omega) < \omega + b$ that leaves the agent indifferent but increases her own payoff. However, proving this result requires a more subtle argument, since incentive compatibility requires $x(\cdot)$ to be non-decreasing. We do this in lemma 4.3.

Lemma 4.3. *In the solution to (PP), (i) $x(\omega) \leq \omega + b$ for all $\omega \in [0,1]$, and (ii) for any $\omega, \omega' \in \Omega$ such that $\omega' > \omega$, then $m(\omega') \leq m(\omega)$.*

We are now ready to characterize the solution to (PP). Substituting (8) into the objective function (PP), the FOC with respect to m_0 is

$$E\left[V_m(x(\omega), m(\omega)|\omega)\right] = 0. \tag{9}$$

[20] Incentive compatibility requires only that $x(\cdot)$ is weakly increasing. However, in a separating contract $x(\cdot)$ must be strictly increasing, for if $x'(\cdot) = 0$ in an interval $[a,b] \subset [0,1]$, then (7) implies that $m'(\cdot) = 0$ in $[a,b]$, which implies pooling.

Equation (9) says that the value for the legislator of increasing the policy budget has to be zero *on average*. Thus, the value of overfunding in some states (with respect to her first best) has to equate, in expectation, the value of underfunding in other states. Given a policy function $x(\cdot)$, this condition pins down m_0.

Now consider the $x(\omega)$-FOCs. At any point ω such that $V_m(x(\omega), m(\omega)|\omega) \neq 0$, the FOC with respect to $x(\omega)$ can be written as

$$\frac{V_x(\cdot|\omega)}{V_m(\cdot|\omega)} = \frac{U_x(\cdot|\omega)}{U_m(\cdot|\omega)} + \beta\left(\frac{F(\omega)}{f(\omega)}\frac{E\left[V_m(\cdot)|\omega' \leq \omega\right]}{V_m(\cdot|\omega)}\right). \tag{10}$$

On the other hand, at any point ω in which the scale of implementation coincides with the legislator's first best, and thus $V_m(x(\omega), m(\omega)|\omega) = 0$, we must have

$$V_x(x(\omega), m(\omega)|\omega) = \beta\frac{F(\omega)}{f(\omega)}E\left[V_m(x(\omega'), m(\omega')|\omega')|\omega' \leq \omega\right] \tag{11}$$

Together with (8), equations (9), (10), and (11) completely characterize the solution to the legislator's problem. We now use these conditions to study the nature of the distortions in the optimal incentive compatible plan with respect to the legislator's first best.

In Theorem 4.5, we characterize distortions in the scale of implementation. We show that in the optimal incentive compatible plan, there is a threshold $\omega^* \in (0, 1)$ such that the scale of implementation is above (below) the legislator's first best in states in which the legislator's preferred policy is smaller (larger) than the threshold. In the context of our running example, the state space can be partitioned into a "low" and a "high" set of states, such that in any state ω in the low (high) set, the bailout is larger (smaller) than what Congress would prefer to implement.

The logic is as follows. If the scale of implementation is above the legislator's preferred level for all realizations of the state, then the legislator's marginal value of increasing the policy's budget is negative, and therefore her marginal value of increasing scale is negative on average. This contradicts the necessary condition (9) for optimality. By the same argument, we cannot have a scale of implementation below the legislator's ideal for all realizations of the state. It follows that in the optimal plan, there must be a (positive measure) set of states in which the legislator overfunds the agency, and one in which the legislator underfunds the agency. Moreover, we know from our previous argument that the scale of implementation $m(\cdot)$ is non-increasing in the state. Thus, if there is overfunding for some state ω, there will optimally be overfunding for all states $\omega' < \omega$. Similarly, if there is underfunding for some state ω, there will optimally be underfunding for all states $\omega' > \omega$.

The previous argument shows that if $m(\cdot)$ crosses $\hat{m}$ at a single point, there is a cutoff ω^* such that the optimal policy involves overfunding for states below ω^*, and underfunding for states above ω^*. Alternatively, there is the possibility that $m(\omega) = \hat{m}$ for all ω in an interval $I \subset \Omega$. By (7), this could only happen if $x(\omega) = \omega + b$ for all $\omega \in I$. In our next lemma we rule this out more broadly. We show that, generically, there is no interval $I' \subset \Omega$ in which $x(\omega) = \omega + b$ for all $\omega \in I'$, independently of the level of m. Thus, in the optimal mechanism, policy is below the agent's preferred policy, except possibly for a set of zero measure.

Lemma 4.4. *Generically, $x(\omega) < \omega + b$ almost everywhere.*

We can now state the first major result of this section.

Theorem 4.5. *Suppose that there is no interval $I \subset \Omega$ such that $f'(\omega) = 0$ for all $\omega \in I$, or that $f = U[0, 1]$. There exists a $\omega^* \in (0, 1)$ such that $m(\omega^*) = \hat{m}$ and (i) the scale of implementation is above the legislator's first best, $m(\omega) > \hat{m}$, for all $\omega < \omega^*$, and (ii) below the legislator's first best, $m(\omega) < \hat{m}$, for all $\omega > \omega^*$.*

Using equation (10), we can look at this result from a different perspective. Note that in the planner's problem,

$$\frac{V_x(\cdot|\omega)}{V_m(\cdot|\omega)} = \frac{U_x(\cdot|\omega)}{U_m(\cdot|\omega)} \quad \forall \omega \in [0,1]$$

Now, Theorem 4.5 implies that $V_m(\cdot|\omega) < 0$ for all $\omega < \omega^*$ and $V_m(\cdot|\omega) > 0$ for all $\omega > \omega^*$. Then (10) implies that

$$\frac{V_x(\cdot|\omega)}{V_m(\cdot|\omega)} \begin{cases} > \frac{U_x(\cdot|\omega)}{U_m(\cdot|\omega)} & \text{if } \omega \in (0,\omega^*) \\ < \frac{U_x(\cdot|\omega)}{U_m(\cdot|\omega)} & \text{if } \omega \in (\omega^*,1) \end{cases} \tag{12}$$

Equation (12) says that the legislator's relative valuation of increases in policy relative to budget is inefficiently larger than that of the agent for low ω, and inefficiently lower than that of the agent for high ω. Thus, absent incentive considerations, for any $\omega \in (0,1)$ there is an alternative plan $(x'(\omega), m'(\omega))$ to $(x(\omega), m(\omega))$ that would make both legislator and agent better off.

Consider first $\omega < \omega^*$. If $x \in (\omega, \omega + b)$ and $m > \hat{m}$, so that the agent and legislator have conflict of interests in both dimensions, both legislator and agent could gain from a trade in which policy moves towards the preferred policy for the legislator (lower x), increasing the agency's budget. If instead the agent's and legislator's interests are aligned in at least one dimension (say for example $m < \hat{m}$, so that $V_m > 0$ and $U_m > 0$) then there is a move in that dimension that would improve the welfare of both.

Suppose instead $\omega > \omega^*$. If agent and legislator have conflict of interests in both dimensions, the legislator and agent could gain from a trade in which policy moves in the direction of the agent, reducing the agency's budget. As before, if the agent's and legislator's interests are aligned in at least one dimension, there is a move in that dimension that would improve the welfare of both.

Now, from (4), the agent's marginal rate of substitution is $\beta(\omega+b-x)$, and from (3), the legislator's marginal rate of substitution is $\alpha\,(x-\omega)\,/\,(m-\hat{m})$. Substituting in (12), we have that for all $\omega<\omega^*$,

$$x(\omega)-\omega>(\beta/\alpha)(\omega+b-x(\omega))(m(\omega)-\hat{m})$$

Since we have shown that $x(\omega)\leq\omega+b$ for all $\omega\in\Omega$, and that $m(\omega)>\hat{m}$ for all $\omega<\omega^*$, it follows that $x(\omega)>\omega$ for all $\omega<\omega^*$ (the strict inequality follows since $x(\omega)=\omega$ leads to a contradiction). This gives a complete characterization of the distortions for $\omega<\omega^*$, which we state in the following theorem.

Theorem 4.6. *For any $\omega<\omega^*$, the optimal policy lies between the ideal policies of the agent and the legislator, $\omega<x(\omega)<\omega+b$, and the scale of implementation is larger than in the legislator's first best, $m(\omega)>\hat{\omega}$.*

In our example, theorem 4.6 says that whenever the optimal allocation of funds to large firms is lower than the threshold ω^*, the allocation of funds to large firms is larger than the first best for Congress but smaller than the first best for the Treasury, and the overall size of the bailout is larger than in Congress' first best.

A similar argument for $\omega>\omega^*$ does not work. We can show, in fact, that $x(\cdot)$ crosses ω from above at some point $\tilde{\omega}\in(\omega^*,1)$. We then show that the optimal policy $x(\cdot)$ stays below the legislator's first best for all $\omega>\tilde{\omega}$. That is, in states in which the legislator prefers relatively high policies, both policy and scale of implementation are below the first best for both agent and legislator. This gives our third major result.

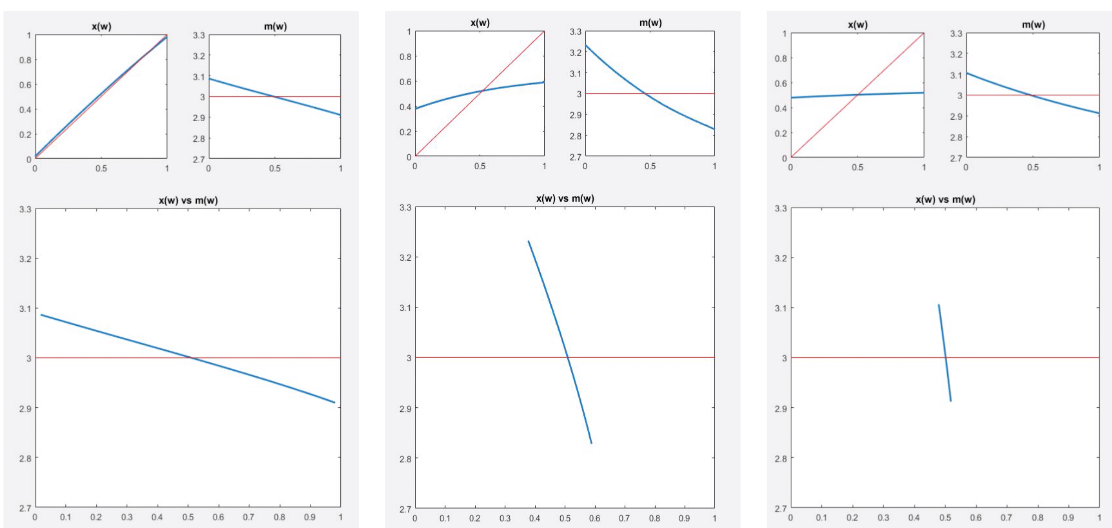
Theorem 4.7. *There exists a non-empty interval* $[\omega^*, \tilde{\omega}]$*, where* $0 < \omega^* < \tilde{\omega} < 1$*, in which the optimal incentive compatible policy lies between the preferred policies of the agent and the legislator, but the scale of implementation is inefficiently low, below the legislator's and the agent's first best. For all states above* $\tilde{\omega}$*, instead, both policy and scale of implementation are inefficiently low, below the first best for both the agent and the legislator.*

Between ω^* and $\tilde{\omega}$, the equilibrium allocation of funds to large firms is larger than the first best for Congress, but smaller than the first best for the Treasury, and the overall size of the bailout is smaller than in Congress' first best. For states above $\tilde{\omega}$, instead, both the allocation of funds to large firms and the size of the bailout are inefficiently low, below the first best for both Treasury and Congress.

Together, theorems 4.5, 4.6 and 4.7 provide a a complete characterization of the distortions in the optimal incentive compatible mechanism, relative to the legislator's first best. The optimal incentive compatible mechanism is ex post inefficient in all states $\omega \in (0, 1)$, but the nature of the distortion changes throughout Ω. For simplicity, we describe these in the context of our example.

In states of nature in which Congress would want to focus heavily on small firms, $\omega < \omega^*$, Treasury and Congress have conflicting interests on both dimensions, as $x(\omega) \in (\omega, \omega + b)$ and $m(\omega) > \hat{\omega}$. Still, there are gains from trade ex post. Since $V_x(\cdot|\omega)/V_m(\cdot|\omega) > U_x(\cdot|\omega)/U_m(\cdot|\omega)$ at any $\omega < \omega^*$, both Congress and Treasury could gain from a trade in which small firms receive a larger allocation of funds (reduce x), increasing the size of the bailout. In intermediate states, $\omega \in (\omega^*, \tilde{\omega})$, the optimal incentive compatible policy lies between the preferred policies of Congress and Treasury, but the size of the bailout is inefficiently low, below Congress' and Treasury's first best. Thus, Congress and Treasury have conflict of interests in one dimension (policy), and aligned interests in the second one (size fo the bailout). As a result, both would benefit ex post from increasing the size of the bailout, keeping the allocation constant. In states in which the legislator would want to favor large firms more heavily, $\omega > \tilde{\omega}$, both the allocation of funds to large firms and the overall size of the bailout are inefficiently low, below the first best for both Congress and the Treasury. In this range, Congress and Treasury have aligned interests on both dimensions, and would both benefit ex post from increasing the allocation to large firms and the overall size of the bailout.

Figure 3 plots the optimal mechanism for various levels of the bias parameter, b. The top panels plot the policy function $x(\cdot)$ and the budget function $m(\cdot)$, as a function of the state. The bottom panels plots the $m(\cdot)/x(\cdot)$ graph, which relates the two observables for each value of the state ω.



$b = 0.2$ $b = 2$ $b = 5$

FIGURE 3. Optimal incentive compatible mechanism for various levels of bias, $b = 0.2, 2, 5$. The top panels plot the policy $x(\cdot)$ and budget $m(\cdot)$ as a function of the state ω. The bottom panels plots the $m(\cdot)/x(\cdot)$ graph (m on the y-axis).

For low conflict of interests, policy is close to the legislator's preferred policy for all realizations of ω, and the budget function remains close to the legislator's first best budget, $\hat{m}$. As b increases, however, the distortions with respect to the legislator's first best increase. The policy function becomes less responsive to the state, and there are larger changes in scale of implementation, leading to large welfare losses for high states. Thus, while for low conflict of interests most of the variability in outcomes is captured in the policy, for moderately high conflict of interests the scale of implementation (e.g., the size of the bailout) becomes inefficiently responsive to the state.

As the conflict of interests between the legislator and the agent increases even more (to $b = 5$ in the figure), there is an almost complete lack of freedom afforded to the agency—very tight budget control and very little policy discretion—as we approach a pooling outcome. Note, in particular, that the range of possible budget realizations becomes small, as in the case of the low bias level. Figure 4 shows this non-monotonicity in the budget range for a specific set of parameters.

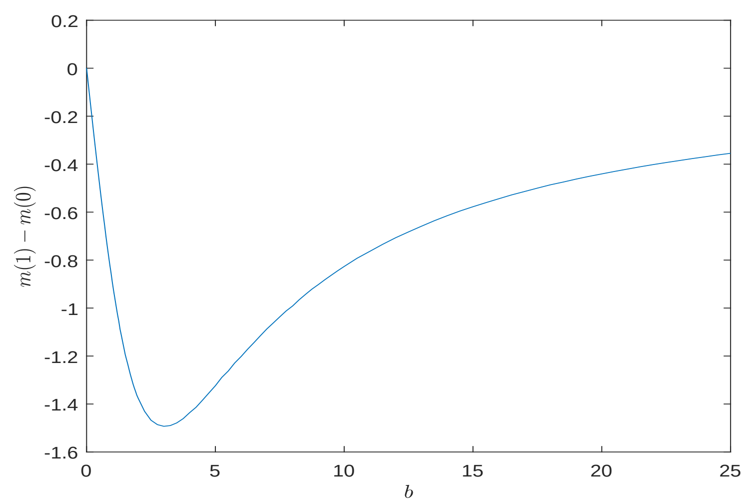


FIGURE 4. The range of the budget across all states, $|m(1) - m(0)|$, is non-monotonic with respect to policy bias, b. In this example, we fix $\alpha = 9$ and $\beta = 1$.

4.4. **Optimality of the Separating Contract.** In this section, we provide conditions for the solution to the legislator's problem to be fully separating, as we assumed in Section 4.3. In particular, we show that provided that the value the legislator gives to policy relative to scale of implementation (α) is large enough relative to the value the agent gives to policy relative to scale of implementation (β), the solution to the legislator's problem is fully separating, independently of the value of the bias, b.

We begin with a more intuitive, limiting argument. Note that (10) can be written as

$$x(\omega) - \omega = \frac{b - \frac{[1-F(\omega)]}{f(\omega)}\left(\frac{E[V_m(\cdot)|\omega' \geq \omega]}{V_m(\cdot)}\right)}{\left[1 + \frac{\alpha}{\beta}\frac{1}{(m(\omega)-\hat{m})}\right]} \tag{13}$$

where $m(\cdot)$ is given by (8). Since the right hand side goes to zero as $\beta \to 0$, it follows that in any interval in which the solution is differentiable, $x(\omega) \to \omega$ as $\beta \to 0$. But then given weak monotonicity and no jumps, there is no pooling in any interval $I \subset \Omega$. A standard delta-epsilon argument shows that the same result holds for sufficiently low β.

It is also straightforward to show that if $\beta b < \sqrt{\alpha}$, there is a (suboptimal) incentive compatible separating contract that dominates a fully uncontingent plan (full pooling).

Remark 4.8. *If $\beta b < \sqrt{\alpha}$, a fully uncontingent plan (x^p, m^p) is not optimal.*

We now go more in depth to explore conditions for a fully separating contract to be optimal, away from the limit. In doing this, we assume that $\omega \sim U[0,1]$ for simplicity. We call the legislator's problem (including the $x' \geq 0$ constraint) the screening problem. This always has a solution. We call the relaxed problem without the $x' \geq 0$ constraint the optimal control problem. The solution to the screening problem coincides with the solution to the optimal control problem if $x(\cdot)$ is weakly increasing. We begin by assuming that in the solution, $x(1) \geq 1 - \frac{b}{2}$ (we refer to this as **Assumption 1**). After the main result, we derive simple conditions on the primitives of the model which guarantee that his assumption holds.[21]

In Lemma B.2.3 we show that in the solution to the legislator's problem, $x(\cdot)$ satisfies

$$x'(\omega) = \frac{\alpha + 2\beta(m - \hat{m}) + \beta V_x \int_0^{\omega} V_m(\omega') \, \mathrm{d}\omega'}{\alpha + \beta(m - \hat{m}) + \beta^2(\omega + b - x)^2}. \tag{14}$$

We will use the above expression to show conditions under which a separating equilibrium exists. We first state a result which is the key step in the proof of the main result of this section.

Lemma 4.9. *If $x'(\omega) = 0$ then $x''(\omega) \leq 0$.*

The above lemma says that if x is non-increasing for some ω, it must be non-increasing for all larger ω. We use the converse to prove the main theorem. In particular, if x is increasing at $\omega = 1$ then it must be increasing for all other $\omega \in [0, 1)$.

Theorem 4.10. *Under Assumption 1, we have that $x'(\omega) > 0$ for all $\omega \in \Omega$; i.e., a fully separating solution to the legislator's screening problem.*

We now give sufficient conditions for Assumption 1 in terms of the primitives of the model. Observe that if the legislator is going to pool all ω together, then $x(1) = \frac{1}{2}$ (if only higher ω are being pooled, $x(1)$ can only be higher). Thus in any solution to the legislator's problem, Assumption 1 is satisfied if $b \geq 1$, for any $\beta \geq 0$. On the other hand, as we showed above, if β is sufficiently low, the agent puts a large weight on the budget allocation, m, relative to the policy outcome x. Then the optimal policy

[21]It is straightforward to prove that $x(1) > 1 - b$ is necessary for a fully separating solution to the screening problem (see the last step of the proof of theorem 4.10), so that assumption 1 is not much stronger than what is necessary.

will be close to the legislator's first-best policy outcome, and as a result $x(1)$ will be close to 1.

Lemma 4.11. *If $\beta^2 < 8\alpha/27$ then assumption 1 is also satisfied for all $b > 0$.*

5. Conclusion

In this paper, we reconsider the problem of congressional control of the bureaucracy for the cases in which the policy under consideration has a budgetary component. Such agencies, we argue, are pervasive throughout the federal government.

Our approach integrates two literatures that, oddly, have had little overlap. On the one hand, a number of papers have focused solely on the policy dimension. These papers have built on the delegation solution first studied by Holmström (1977). In this context, optimal agency control boils down to a range of policies in which the agent is left discretion. On the other hand, earlier papers on congressional control of the bureaucracy focused solely on budgets, with agencies producing a quantity of "output" (Niskanen (1971), Miller and Moe (1983), Bendor, Taylor, and Van Gaalen (1985, 1987), Banks (1989)). We argue that the budgets associated to each particular policy can be, and often are, used to affect agencies' policy decisions. With this premise, we take a mechanism design approach, in the spirit of the delegation solution, to characterize the optimal institutional arrangement for the legislature in this setting.

We show that, when the scale of implementation matters, the optimal incentive compatible policy is below the agent's preferred policy almost everywhere. Furthermore, the optimal separating mechanism entails reducing the budget for policies that go further in the direction of the agent's preferences; even more, in some states, than what the legislator herself would want.

In solving the model, we focus on direct truthful mechanisms. While our solution is silent about the details of any indirect mechanism that the legislator might be using, the empirical implications for budgets and policy apply to those as well, by virtue of the revelation principle. In particular, we expect that as the conflict of interest between the legislator and the agency grow, policy will become less responsive to the agent's private information, and thus less variable overall. Budgets, on the other hand, can vary non-monotonically with the degree of conflict of interest. An increase from full alignment to a moderate conflict of interest will lead to more variable budgets. However, when the agent's bias is very large, budgets again become relatively unresponsive to the realization of the agent's private information, as we approach a pooling solution.

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Appendix A. Proofs of Main Results

Lemma A.1.1. *In the first best, $x(\omega) \in (\omega, \omega + b)$.*

Proof of Lemma A.1.1. Note that if $x(\omega) > \omega + b$, then $U_x(\cdot|\omega) < 0$ and $V_x(\cdot|\omega) < 0$, which contradict (6). Similarly, if $x(\omega) < \omega$, then $U_x(\cdot|\omega) > 0$ and $V_x(\cdot|\omega) > 0$, again contradicting (6). Moreover, if $x(\omega) = \omega + b$, then $U_x(\cdot|\omega) = 0$ and $V_x(\cdot|\omega) < 0$, while if $x(\omega) = \omega$, then $U_x(\cdot|\omega) > 0$ and $V_x(\cdot|\omega) = 0$. Again, a contradiction. □

Lemma A.1.2. *Suppose $\hat{m} > \beta b^2$. Then for any ω, the policy in the efficient solution is $x^\dagger(\omega) = \omega + \Delta$ for some $\Delta > 0$.*

Proof of Lemma A.1.2. Given (4), the agent's MRS is $\beta(\omega + b - x)$, and given (3), the legislator's MRS is $\alpha\,(x - \omega)\,/\,(m - \hat{m})$. Then condition (5) is

$$\frac{\alpha}{\beta}\left(\frac{x - \omega}{\omega + b - x}\right) = m - \hat{m} \tag{15}$$

If we write out condition (6) and substitute (15), we obtain

$$\left(\frac{2\lambda\alpha}{\beta}\right)^2 \frac{(x(\omega) - \omega)^2}{(\omega + b - x(\omega))^2} = \frac{A - \left(\frac{\alpha}{\beta}\right)^2 \left(\frac{x(\omega)-\omega}{\omega+b-x(\omega)}\right)^2 - \alpha(x(\omega) - \omega)^2}{\hat{m} + \left(\frac{\alpha}{\beta}\right)\left(\frac{x(\omega)-\omega}{\omega+b-x(\omega)}\right) - \beta(\omega + b - x(\omega))^2} \tag{16}$$

The solution to equation (16) pins down an efficient policy $x^\dagger(\omega)$ for any $\omega \in \Omega$. Then substituting in (15) gives the efficient project size

$$m^\dagger(\omega) = \hat{m} + \frac{\alpha}{\beta}\left(\frac{x^\dagger(\omega) - \omega}{\omega + b - x^\dagger(\omega)}\right) \tag{17}$$

Now, set $x(\omega) = \omega + \Delta$ for all ω. Then (16) is

$$R(\Delta) \equiv \left(\frac{2\lambda\alpha}{\beta}\right)^2 \frac{\Delta^2}{(b - \Delta)^2} - \frac{A - \left(\frac{\alpha}{\beta}\right)^2 \left(\frac{\Delta}{b-\Delta}\right)^2 - \alpha(\Delta)^2}{\hat{m} + \left(\frac{\alpha}{\beta}\right)\left(\frac{\Delta}{b-\Delta}\right) - \beta(b - \Delta)^2} = 0 \tag{18}$$

Note that if there is a solution $\Delta^\dagger$ to (18), then the efficient policy is indeed of the form $x(\omega) = \omega + \Delta$ for all ω. Now, given the assumption that $\hat{m} > \beta b^2$,

$$\lim_{\Delta \to 0} R(\Delta) = -\frac{A}{\hat{m} - \beta b^2} < 0$$

Next, note that multiplying and dividing the second term by $(b - \Delta)$, and taking limits,

$$\lim_{\Delta \to b} R(\Delta) = \lim_{\Delta \to b}\left[\left(\frac{2\lambda\alpha}{\beta}\right)^2 \frac{b^2}{(b - \Delta)^2} + \left(\frac{\alpha}{\beta}\right)\left(\frac{b}{b - \Delta}\right)\right] = \infty$$

It follows that there exists a Δ solving (18). $\square$

Proof of Lemma 4.1. Note that incentive compatibility for the agent implies that for any $\omega \in [0,1]$ and $\omega' > \omega$:

$$\mathcal{U}^a(\omega,\omega) \geq \mathcal{U}^a(\omega',\omega)$$
$$\mathcal{U}^a(\omega',\omega') \geq \mathcal{U}^a(\omega,\omega').$$

Substituting:

$$m(\omega) - m(\omega') \geq \frac{\beta}{2}(\omega + b - x(\omega))^2 - \frac{\beta}{2}(\omega + b - x(\omega'))^2 \tag{19}$$

$$m(\omega') - m(\omega) \geq \frac{\beta}{2}(\omega' + b - x(\omega'))^2 - \frac{\beta}{2}(\omega' + b - x(\omega))^2 \tag{20}$$

Adding the two inequalities, and simplifying, we have

$$[x(\omega') - x(\omega)](\omega' - \omega) \geq 0.$$

This shows $x(\cdot)$ is non-decreasing. Substituting in (19), and using that $x(\omega) \leq \omega + b$ implies that $m(\cdot)$ is non-increasing. $\square$

Proof of Proposition 4.2. Given (4), the FOC for truth-telling (7) is

$$m'(\omega) = -\beta(\omega + b - x)x'(\omega) \tag{21}$$

Define $\tilde{m}(x,\omega)$ so that $\tilde{m}(x(\omega),\omega) \equiv m(\omega)$. By (21),

$$\frac{\partial \tilde{m}(x,\omega)}{\partial x} = -\beta(\omega + b - x(\omega))$$

Integrating with respect to x, we obtain

$$\tilde{m}(x,\omega) = \frac{\beta}{2}(x)^2 - \beta(\omega + b)x + C$$

where the constant of integration C can be a function of ω. Then

$$m(\omega) = \tilde{m}(x(\omega),\omega) = \frac{\beta}{2}(x(\omega))^2 - \beta(\omega + b)x(\omega) + C(\omega) \tag{22}$$

Taking the derivative with respect to ω,

$$m'(\omega) = -\beta x'(\omega)[\omega + b - x(\omega)] - \beta x(\omega) + C'(\omega) \tag{23}$$

From (21) and (23), we need $C'(\omega) = \beta x(\omega)$, so $C(\omega) = \beta \int_0^\omega x(r)dr + \kappa$. Then, substituting in (22), we get an expression of $m(\cdot)$ in terms of ω and $x(\omega)$:

$$m(\omega) = \frac{\beta}{2}(x(\omega))^2 - \beta(\omega + b)x(\omega) + \beta\int_0^\omega x(r)dr + \kappa,$$

which is (8). Finally, evaluating at $\omega = 0$, we get

$$\kappa(m_0, x_0) = m_0 - \frac{\beta}{2}(x_0)^2 + \beta b x_0$$

□

Remark A.1.3. *The Hamiltonian for problem (PP) is*

$$\mathcal{H} = U^p(x, m|\omega) f(\omega) - \lambda_1 \frac{U_x^a(x, m|\omega)}{U_m^a(x, m|\omega)} y + \lambda_2 y$$

The necessary and sufficient conditions for a fully separating solution are that there exist $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ such that:

$$m' = \mathcal{H}_{\lambda_1} = -\frac{U_x^a(x, m|\omega)}{U_m^a(x, m|\omega)} y \tag{24}$$

$$x' = \mathcal{H}_{\lambda_2} = y \tag{25}$$

$$0 = \mathcal{H}_y = -\lambda_1 \frac{U_x^a(x, m|\omega)}{U_m^a(x, m|\omega)} + \lambda_2 \tag{26}$$

$$\lambda_1' = -\mathcal{H}_m = -U_m^p(x, m|\omega) f(\omega) + \lambda_1 y \frac{\partial}{\partial m}\left(\frac{U_x^a(x, m|\omega)}{U_m^a(x, m|\omega)}\right) \tag{27}$$

$$\lambda_2' = -\mathcal{H}_x = -U_x^p(x, m|\omega) f(\omega) + \lambda_1 y \frac{\partial}{\partial x}\left(\frac{U_x^a(x, m|\omega)}{U_m^a(x, m|\omega)}\right), \tag{28}$$

$$0 = \mu m, \tag{29}$$

with initial conditions $m(0) = m_0$ and $x(0) = x_0$ and transversality conditions $\lambda_1(1) = 0$ and $\lambda_2(1) = 0$, and $\lambda_1(0) = 0$ and $\lambda_2(0) = 0$. From the Pontryagin Maximum Principle (for example, see Zeidler (1985) Theorem 48.C), any optimum for the legislator satisfies the Euler-Lagrange equations above. Moreover, the optimal control problem in equations (24-29) satisfies the weak Mangasarian sufficient condition for a maximum (the problem is in general weakly concave), and thus a solution to (24-29) is a global maximizer.

Proof of Lemma 4.3. The proof follows a similar argument in Krishna and Morgan (2008), and builds on Remark A.1.3. Suppose, to the contrary, that there exists an ω such that $x(\omega) > b + \omega$. Consider (26). Since $x(\omega) > b + \omega$, we have $U_x^a(x, m|\omega) < 0$. Suppose first that $\lambda_1 > 0$. Since $U_m^a(x, m|\omega) > 0$ and $\lambda_2 \geq 0$, we have

$$-\lambda_1 \frac{U_x^a(x, m|\omega)}{U_m^a(x, m|\omega)} + \lambda_2 > 0$$

which is a contradiction (this expression has to equal zero by (26)). Suppose then that $\lambda_1 = 0$. Then from (28)

$$\lambda_2' = -U_x^p(x, m|\omega) f(\omega) > 0 \Rightarrow \lambda_2(\omega) > 0,$$

which again contradicts (26). □

Proof of Lemma 4.4. Assume by way of contradiction that there exists some interval $I \subset \Omega$ such that $x(\omega) = \omega + b$ for all $\omega \in I$. Then by (7) we have that $m'(\omega) = 0$ for all $\omega \in I$, i.e., $m(\omega) = m_c$. Thus at any $\omega \in I$,

$$V_m(\cdot|\omega) = (m_c - \hat{m})/\sqrt{A - (m_c - \hat{m})^2 - \alpha b^2} \equiv k_0,$$

and

$$V_x(\cdot|\omega) = -(\alpha b)/\sqrt{A - (m_c - \hat{m})^2 - \alpha b^2} \equiv k_1.$$

Suppose first that $m_c \neq \hat{m}$. Then from (10), noting that $U_x/U_m = 0$ since $x(\omega) = \omega + b$, for any $\omega \in I$,

$$\begin{aligned} V_x(x(\omega), m(\omega)|\omega) &= \beta \frac{1}{f(\omega)} \int_0^\omega V_m(x(\omega'), m(\omega')|\omega') f(\omega') d\omega' \\ f(\omega) &= \frac{\beta}{k_1} \int_0^\omega V_m(x(\omega'), m(\omega')|\omega') f(\omega') d\omega' \\ f'(\omega) &= \beta \frac{k_0}{k_1} = -\beta \frac{m_c - \hat{m}}{\alpha b} \end{aligned}$$

We note that this condition on the derivative of the PDF f is non-generic in the appropriate space of probability measures, see footnote 11.

If $m_c = \hat{m}$, we obtain the same expression using (11). This still allows the case $\omega \sim U[0, 1]$. But here we have

$$\begin{aligned} V_x(x(\omega), m(\omega)|\omega) &= \beta \frac{F(\omega)}{f(\omega)} E\left[V_m(x(\omega'), m(\omega')|\omega')|\omega' \leq \omega\right] \\ k_1 &= \beta \omega E\left[V_m(x(\omega'), m(\omega')|\omega')|\omega' \leq \omega\right] \end{aligned}$$

Since $V_m(\cdot|\omega) < 0$ for $\omega < \min I$ and $V_m(\cdot|\omega) = 0$ for all $\omega \in I$, $E\left[V_m(\cdot)|\omega'' \leq \omega\right]$ is increasing in ω. It follows that the RHS is increasing in ω, leading to a contradiction. □

Proof of Theorem 4.5. Suppose $m(\omega) \geq \hat{m}$ for all $\omega \in [0, 1]$. Note that then $V_m(\cdot|\omega) < 0\ \forall \omega \in [0, 1]$. But then (9) cannot hold. Similarly, if $m(\omega) \leq \hat{m}$ for all $\omega \in [0, 1]$, then $V_m(\cdot|\omega) > 0\ \forall \omega \in [0, 1]$, but then (9) cannot hold either. We know already that if $\beta > 0$, in the solution we cannot have $m(\omega) = \hat{m}$ for all $\omega \in [0, 1]$. It follows that there is a positive measure set of $[0, 1]$ for which $m(\omega) > \hat{m}$, and a positive measure set for which $m(\omega) < \hat{m}$. Now, we have shown that $m'(\omega) \leq 0$. Thus $m(\cdot)$ can only cross $\hat{m}$ from above.

There are now two possibilities: either (i) $m(\cdot)$ crosses $\hat{m}$ and there exists a ω^* such that $m(\omega) \geq \hat{m}$ for all $\omega \leq \omega^*$ and $m(\omega) \leq \hat{m}$ for all $\omega \geq \omega^*$, or (ii) there exist $(\underline{\omega}, \overline{\omega})$, with $0 < \underline{\omega} < \overline{\omega} < 1$, such that $m(\omega) > \hat{m}$ for all $\omega < \underline{\omega}$ and $m(\omega) < \hat{m}$ for all $\omega > \overline{\omega}$, with $m(\omega) = \hat{m}$ for all $\omega \in [\underline{\omega}, \overline{\omega}]$ (note that in this case we would need $0 < \underline{\omega}$ and $\overline{\omega} < 1$, for otherwise we would violate the zero expected marginal condition). Then from (7), is has to be that $x = \omega + b$ for all $\omega \in [\underline{\omega}, \overline{\omega}]$. But lemma 4.4 shows that this cannot happen. ☐

Proof of Theorem 4.7. Suppose that $x(\omega) > \omega$ for all $\omega > \omega^*$. Then $\frac{V_x(\cdot|\omega)}{V_m(\cdot|\omega)} < 0$ and $\frac{U_x(\cdot|\omega)}{U_m(\cdot|\omega)} > 0$, so we must have $\beta\left(\frac{F(\omega)}{f(\omega)}\frac{E[V_m(\cdot)|\omega' \leq \omega]}{V_m(\cdot|\omega)}\right) < 0$ for all such ω. But at $\omega = 1$, $\left(\frac{F(1)}{f(1)}\frac{E[V_m(\cdot|\omega')|\omega' \leq 1]}{V_m(\cdot|1)}\right) = 0$, a contradiction.

It follows that at some point $\tilde{\omega}$, $x(\cdot)$ crosses ω. At that point, $V_x(\cdot|\tilde{\omega}) = 0$, $V_m(\cdot|\tilde{\omega}) > 0$, $U_x(\cdot|\tilde{\omega}) > 0$ and $U_m(\cdot|\tilde{\omega}) > 0$ so that

$$\beta\left(\frac{[1 - F(\tilde{\omega})]}{f(\tilde{\omega})}\frac{E\left[V_m(\cdot)|\omega' \geq \tilde{\omega}\right]}{V_m(\cdot|\tilde{\omega})}\right) = \frac{U_x(\cdot|\tilde{\omega})}{U_m(\cdot|\tilde{\omega})} > 0$$

and

$$-\beta\left(\frac{F(\tilde{\omega})}{f(\tilde{\omega})}\frac{E\left[V_m(\cdot)|\omega' \leq \tilde{\omega}\right]}{V_m(\cdot|\tilde{\omega})}\right) = \frac{U_x(\cdot|\tilde{\omega})}{U_m(\cdot|\tilde{\omega})} > 0$$

From the first expression, it follows that $E\left[V_m(\cdot)|\omega' \geq \tilde{\omega}\right] > 0$, and from the second, that $E\left[V_m(\cdot)|\omega' \leq \tilde{\omega}\right] < 0$. It follows that $\tilde{\omega} < 1$. Moreover, we must have $\tilde{\omega} > \omega^*$. For suppose $\tilde{\omega} = \omega^*$. Then $V_x(\cdot|\omega^*) = V_m(\cdot|\omega^*) = 0$, so that (11) becomes

$$0 = E\left[V_m(x(\omega'), m(\omega')|\omega')|\omega' \geq \omega^*\right],$$

which is impossible given (9) and $m(\omega) > \hat{m}$ for all $\omega < \omega^*$.

Now, we want to show that $x(\cdot)$ can only cross ω once. Suppose it crosses twice, at points $\tilde{\omega}$ and $\tilde{\omega}'$. Then from 13,

$$b = -\frac{F(\tilde{\omega})}{f(\tilde{\omega})}\left(\frac{E\left[V_m(\cdot)|\omega' \leq \tilde{\omega}\right]}{V_m(\cdot)}\right)$$

and

$$b = -\frac{F(\tilde{\omega}')}{f(\tilde{\omega}')}\left(\frac{E\left[V_m(\cdot)|\omega' \leq \tilde{\omega}'\right]}{V_m(\cdot)}\right),$$

but this is impossible since the right hand side is, as we have shown before, decreasing in ω. ☐

Proof of Remark 4.8. The payoff of the legislator in a pooling contract is (using $m^p = \hat{m}$),

$$\int_0^1 V(x^p, m^p|\omega)f(\omega)d\omega = \int_0^1 \left[A - \alpha(\omega - x^p)^2\right]^{1/2} f(\omega)d\omega$$

The FOC is

$$\int \frac{\omega - x^p}{A - \alpha(\omega - x^p)} f(\omega) d\omega = 0.$$

This gives the optimal x^p.

Now consider a separating contract in which the the policy function is the legislator's first best, $x(\omega) = \omega$. From (8), then $m(\omega) = m_0 - \beta b\omega$. This gives the legislator a payoff

$$\int V(x, m|\omega) f(\omega) d\omega = \int \left[A - (m_0 - \beta b\omega - \hat{m})^2\right]^{1/2} f(\omega) d\omega$$

This is better than pooling iff

$$\int \left\{ \left[A - (m_0 - \beta b\omega - \hat{m})^2\right]^{1/2} - \left[A - \alpha(\omega - x^p)^2\right]^{1/2} \right\} f(\omega) d\omega \geq 0$$

Now, one can do better, but can certainly choose $m_0 = \beta b x^p + \hat{m}$. With this, the separating payoff would be

$$\int \left\{ \left[A - \beta^2 b^2 (\omega - x^p)^2\right]^{1/2} - \left[A - \alpha(\omega - x^p)^2\right]^{1/2} \right\} f(\omega) d\omega \geq 0$$

so the suboptimal separating contract is better than pooling if $\beta b < \sqrt{\alpha}$. $\square$

Appendix A.2. Online Appendix: Additional Proofs

Contents

B.2.1. **Proofs for Section 3.**

Proof of Proposition 3.1. The intuition for the result is illustrated in Figure B.2.1. The legislator's indifference curves in state 0 and 1 are depicted in blue and red, respectively. The set of points where the indifference curves in the two states are tangent to one another is shown by the green line. Note that if an optimal pooling contract (x_p^*, m_p^*) is proposed, it will be somewhere on this line, for otherwise we can improve the legislator's utility by proposing a pooling contract in this set. (In particular, the optimal pooling contract for $f(0) = 2/5$ is shown by the black circle.) Note however that if (x_p^*, m_p^*) is an optimal contract, it must be that the agent's indifference curve in state 0 is also tangent to the legislator's indifference curves at this point. Otherwise utility can be improved by moving "inside" the legislator's better-than sets in each state, as shown by the black triangles in the figure. It follows that a pooling contract (x_p^*, m_p^*) can only be optimal if a triple tangency of indifference curves is satisfied, a property that only holds in a closed nowhere-dense set of utility functions.

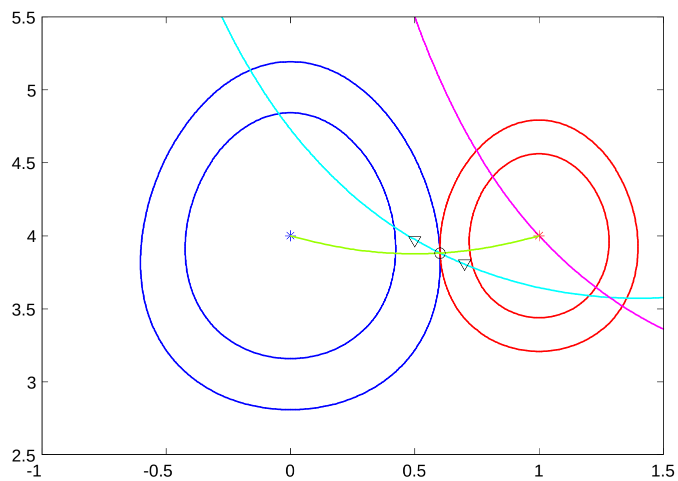

FIGURE B.1. Agency Discretion: Pooling Contracts are Not Optimal

We now show this result formally. We will show that the utility functions which satisfy a necessary condition for pooling are nowhere dense in the appropriate Sobolev space, $W^{1,p}(X)$. In particular, we show that if a pooling contract x_o^*, m_o^* is optimal for the legislator, then the following triple tangency condition must be satisfied:

$$\gamma'_{p,0}\left(t^*_{p,0}\right) = \pm\gamma'_{p,1}\left(t^*_{p,1}\right) = \pm\gamma'_{a,0}\left(t^*_{a,0}\right),$$

where $\gamma_{j,\omega} : I \to \mathbb{R}^2$ is the parametrization by arclength of $IC^j\left(x_o^*, m_o^* | \omega\right)$, $\gamma_{j,\omega}\left(t^*_{j,\omega}\right) = \left(x_o^*, m_o^*\right)$ and $I \subset \mathbb{R}$ is a non-empty interval.

First, the legislator's indifference curves have to be tangent, for otherwise (x_o^*, m_o^*) does not solve the optimal pooling problem,

$$\max_{(x_o,m_o)} \sum_{\omega\in\{0,1\}} f(\omega)V(x_o, m_o|\omega).$$

To see this, note that the first-order condition of the above problem implies:

$$\nabla V(x_o^*, m_o^*|0) = \frac{-f_1}{f_0}\nabla V(x_o^*, m_o^*|1).$$

It follows that for any (x, m):

$$\begin{aligned} \nabla V(x_o^*, m_o^*|0)\cdot[(x,m)-(x_o^*, m_o^*)] &= 0 \\ \Leftrightarrow \nabla V(x_o^*, m_o^*|1)\cdot[(x,m)-(x_o^*, m_o^*)] &= 0. \end{aligned}$$

Now, by definition of $\gamma_{j,\omega}$:

$$\begin{aligned} \nabla V(x_o^*, m_o^*|0)\cdot\left[\gamma'_{p,0}\left(t^*_{p,0}\right)-(x_o^*, m_o^*)\right] &= 0, \text{ and} \\ \nabla V(x_o^*, m_o^*|1)\cdot\left[\gamma'_{p,1}\left(t^*_{p,1}\right)-(x_o^*, m_o^*)\right] &= 0, \end{aligned}$$

and since $\left\|\gamma'_{p,1}\left(t^*_{p,1}\right)\right\| = \left\|\gamma'_{p,0}\left(t^*_{p,0}\right)\right\| = 1$ (because $\gamma_{j,\omega}$ is the natural parametrization), it follows that $\gamma'_{p,0}\left(t^*_{p,0}\right) = \pm\gamma'_{p,1}\left(t^*_{p,1}\right)$.[22] This proves the first equality.

We will show the second equality by contradiction. Assume that $\gamma'_{p,1}\left(t^*_{p,1}\right) \neq \pm\gamma'_{a,0}\left(t^*_{a,0}\right)$. Note that for any ε, the menu $\left(\varepsilon\gamma'_{a,0}\left(t^*_{a,0}\right), -\varepsilon\gamma'_{a,0}\left(t^*_{a,0}\right)\right)$ is incentive compatible[23], since both contracts are on $IC^a\left(x_o^*, m_o^*|0\right)$ [alternatively, could move along IC curve by proposing menu $\left(\gamma_{a,0}\left(t^*_{a,0}-\varepsilon\right), \gamma_{a,0}\left(t^*_{a,0}+\varepsilon\right)\right)$]. Because $\gamma'_{p,1}\left(t^*_{p,1}\right) \neq \pm\gamma'_{a,0}\left(t^*_{a,0}\right)$, either

$$\varepsilon\gamma'_{a,0}\left(t^*_{a,0}\right) \in intB^p\left(x_o^*, m_o^*|0\right), -\varepsilon\gamma'_{a,0}\left(t^*_{a,0}\right) \in intB^p\left(x_o^*, m_o^*|1\right)$$

or

$$\varepsilon\gamma'_{a,0}\left(t^*_{a,0}\right) \in intB^p\left(x_o^*, m_o^*|1\right), -\varepsilon\gamma'_{a,0}\left(t^*_{a,0}\right) \in intB^p\left(x_o^*, m_o^*|0\right).$$

Without loss of generality, assume the first holds. This implies that

$$V(\varepsilon\gamma'_{a,0}\left(t^*_{a,0}\right)|0) > V(x_o^*, m_o^*|0) \quad\text{and}\quad V(-\varepsilon\gamma'_{a,0}\left(t^*_{a,0}\right)|1) > V(x_o^*, m_o^*|1),$$

thus the separating menu dominates the pooling menu state by state, which contradicts the optimality of pooling. □

Lemma B.2.1. $(x^*(\omega), m^*(\omega)) = (\hat{x}_\omega, \hat{m}_\omega)$ *for* $\omega \in \{0,1\} \Leftrightarrow b \leq 1/2$.

[22]Note that the sign depends on the direction of the parametrization; reversing the direction would change the sign.

[23] Note that we are using the convention in differential geometry that $\gamma'_{j,\omega}(t)$ is a vector with base point at $\gamma_{j,\omega}(t)$ and that $\varepsilon\gamma'_{j,\omega}(t)$ is a tangent vector length ε, instead of the unit tangent vector (this is a slight abuse of notation).

Proof of Lemma B.2.1. The Lagrangian for the legislator is:

$$\sum_{\omega} V(x_\omega, m(\omega)|\omega) f(\omega) + \lambda_0 \left[U(x(0), m(0)|0) - U^a(x(1), m(1)|0)\right]$$
$$+\lambda_1 \left[U(x(1), m(1)|1) - U(x(0), m(0)|1)\right]$$

The first order conditions are:

$$V_x(x(0), m(0)|0) f(0) + \lambda_0 U_x(x(0), m(0)|0) - \lambda_1 U_x(x(0), m(0)|1) = 0, \tag{30}$$

$$V_x(x(1), m(1)|1) f(1) - \lambda_0 U_x(x(1), m(1)|0) + \lambda_1 U_x(x(1), m(1)|1) = 0, \tag{31}$$

$$V_m(x(0), m(0)|0) f(0) + \lambda_0 U_m(x(0), m(0)|0) - \lambda_1 U_m(x(0), m(0)|1) = 0, \tag{32}$$

$$V_m(x(1), m(1)|1) f(1) - \lambda_0 U_m(x(1), m(1)|0) + \lambda_1 U_m(x(1), m(1)|1) = 0, \tag{33}$$

$$U(x(0), m(0)|0) - U^a(x(1), m(1)|0) \geq 0, \quad \lambda_0 \geq 0, \quad \lambda_0 \frac{\partial \mathcal{L}}{\partial \lambda_0} = 0, \tag{34}$$

$$U(x(1), m(1)|1) - U(x(0), m(0)|1) \geq 0, \quad \lambda_1 \geq 0, \quad \lambda_1 \frac{\partial \mathcal{L}}{\partial \lambda_1} = 0, \tag{35}$$

Suppose that neither constraint is binding. Then we have $\lambda_0^* = \lambda_1^* = 0$. Thus (30) becomes $V_x(x(0), m(0)|0) = 0 \Leftrightarrow x^*(0) = 0$, and (31) becomes $V_x(x(1), m(1)|1) = 0 \Leftrightarrow x^*(1) = \hat{x}_1 = 1$. Then (32) becomes $V_m(x(0), m(0)|0) = 0$, and since $x(0) = 0$, then $V_m(0, m(0)|0) = 0$. Therefore $m(0) = \hat{m}(0)$. Similarly, from (33), $m(1) = \hat{m}(1)$. Then (34) and (35) are reduced to:

$$U\left(b^2, \hat{m}(0)\right) \geq U\left((1-b)^2, \hat{m}(1)\right),$$

and

$$U\left(b^2, \hat{m}(0)\right) \geq U\left((1+b)^2, \hat{m}(1)\right).$$

This concludes the proof of the lemma. □

Lemma B.2.2. *The optimal incentive compatible policy for the legislator lies on the contract curves; i.e.,* $(x^*_\omega, m^*_\omega) \in CC(\omega)$*, and thus*

$$\frac{U^p_x(x^*_\omega, m^*_\omega|\omega)}{U^p_m(x^*_\omega, m^*_\omega|\omega)} = \frac{U^a_x(x^*_\omega, m^*_\omega|0)}{U^a_m(x^*_\omega, m^*_\omega|0)} \quad \textit{for } \omega = 0, 1. \tag{36}$$

Proof of Lemma B.2.2. Suppose in the solution of the legislator's optimization problem $\lambda_0^* > 0$ and $\lambda_1^* = 0$. Then equations (30) and (32) give

$$\frac{V_x(x(0), m(0)|0)}{V_m(x(0), m(0)|0)} = \frac{U_x(x(0), m(0)|0)}{U_m(x(0), m(0)|0)} \tag{37}$$

so that $(x(0)^*, m(0)^*) \in CC(0)$ Similarly, equations (31) and (33) give:

$$\frac{V_x(x(1), m(1)|1)}{V_m(x(1), m(1)|1)} = \frac{U_x(x(1), m(1)|0)}{U_m(x(1), m(1)|0)}, \tag{38}$$

and thus $(x(1)^*, m(1)^*) \in CC(1)$. □

Proof of Proposition 3.2. Note that since $\lambda_0 > 0$ and $U_m(x(0), m(0)|0) > 0$, then (37) implies that $V_m(x(0), m(0)|0) < 0$. Thus, there is overfunding in state 0; i.e., $m(0) > \hat{m}$. Also, since $V_x(x(0), m(0)|0) \geq 0$ iff $x(0) \leq 0$ and $U_x(x(0), m(0)|0) \geq 0$ iff $x(0) \leq b$, (37) implies that $x(0) \in (0, b)$, so the optimal policy in state 0 distorts in favor of the agent. Consider next expression (38). Note that $\lambda_0 > 0$ and $U_m(x(1), m(1)|0) > 0$ in (38) imply that $V_m(x(1), m(1)|1) > 0$. Thus there is underfunding in state 1; i.e., $m(1) < \hat{m}$. And from the first equality, we have that $V_x(x(1), m(1)|1)$ and $U_x(x(1), m(1)|0)$ have to have the same sign, so either $x(1) < \min\{1, b\}$ or $x(1) > \max\{1, b\}$. So suppose first that $b < 1$. Then either $x(1) < b$ or $x(1) > 1$. However, it cannot be that $x(1) < b$. To see this, note that in this case the symmetric point about the ideal point of the agent $(2b - x(1), m(1))$ would give the agent the same payoff but would increase the utility of the legislator. Thus such $(x(1), m(1)) \notin CC(1)$. It follows that if $b < 1$, then $x(1) > 1$. Suppose next that $b > 1$. Then either $x(1) < 1$ or $x(1) > b$, but by a similar argument as before, it must be that $x(1) < 1$.

Finally, we show that if $f(0) \in (0, 1)$, the optimal incentive compatible solution entails distortions in both states: $(x^*_\omega, m^*_\omega) \neq (\hat{x}_\omega, \hat{m}_\omega)$ for $\omega = 0, 1$. Equivalently, we need to show that if $f(0) \in (0, 1)$, the solution to Problem 2 satisfies $u^* \in (U(\hat{x}_0, \hat{m}_0|0), U(\hat{x}_1, \hat{m}_1|0))$. We will show that if $f(0) \neq 0$ then $u^* < U(\hat{x}_1, \hat{m}_1|0)$. A similar argument proves the opposite direction. Since

$$\frac{\partial}{\partial u} V(\tilde{x}^\omega(u), \tilde{m}^\omega(u)|\omega) = V_x(\tilde{x}^\omega(u), \tilde{m}^\omega(u)|\omega)\tilde{x}^\omega_u(u) + V_m(\tilde{x}^\omega(u), \tilde{m}^\omega(u)|\omega)\tilde{m}^\omega_u(u),$$

and when evaluated at the agency utility corresponding to the ideal point for legislator in state 1 this means that

$$\left.\frac{\partial}{\partial u} V(\tilde{x}^1(u), \tilde{m}^1(u)|1)\right|_{U(\hat{x}_1, \hat{m}_1|0)} = 0.$$

Furthermore

$$\left.\frac{\partial}{\partial u} V(\tilde{x}^0(u), \tilde{m}^0(u)|0)\right|_{U(\hat{x}_1, \hat{m}_1|0)} = V_x(\tilde{x}^0(u), \tilde{m}^0(u)|0)\tilde{x}^0_u(u) + V_m(\tilde{x}^0(u), \tilde{m}^0(u)|0)\tilde{m}^0_u(u).$$

Since we are overfunding in state 0, we have $V_m(\tilde{x}^0(u), \tilde{m}^0(u)|0) < 0$. Also, $V_x(\tilde{x}^0(u), \tilde{m}^0(u)|0) < 0$, since the optimal policy is distorted in favor of the agent. Finally, since the indifference curve moves in the north-east direction, we have that $\tilde{x}^0_u(U(\hat{x}_1, \hat{m}_1|0)) > 0$ and $\tilde{m}^0_u(U(\hat{x}_1, \hat{m}_1|0)) > 0$. All of this implies that:

$$\left.\frac{\partial}{\partial u} V(\tilde{x}^0(u), \tilde{m}^0(u)|0)\right|_{U(\hat{x}_1, \hat{m}_1|0)} < 0,$$

which means utility can be improved by decreasing u if $f(0) \neq 0$; thus $u^* < U(\hat{x}_1, \hat{m}_1|0)$. $\square$

B.2.2. **Proofs for Section 4.4.**

Lemma B.2.3. *In the solution to the legislator's problem, $x(\cdot)$ satisfies*

$$x'(\omega) = \frac{\alpha + 2\beta(m - \hat{m}) + \beta V_x \int_0^\omega V_m(\omega') \, d\omega'}{\alpha + \beta(m - \hat{m}) + \beta^2(\omega + b - x)^2}.$$

Proof of Lemma B.2.3. The FOC (10) can be written as

$$\begin{aligned} \frac{\alpha(x(\omega) - \omega)}{(m(\omega) - \hat{m})} &= \beta(b + \omega - x(\omega)) + \frac{\beta \int_0^\omega V_m(\omega') f(\omega') \, d\omega'}{V_m(\omega) f(\omega)} \\ (x(\omega) - \omega) \frac{\alpha + \beta(m(\omega) - \hat{m})}{\beta(m(\omega) - \hat{m})} &= b + \frac{\int_0^\omega V_m(\omega') f(\omega') \, d\omega'}{V_m(\omega) f(\omega)}, \end{aligned} \tag{39}$$

Now let $h(\omega) \equiv \frac{\int_0^\omega g(\omega') \, d\omega'}{g(\omega)}$, where $g(\omega) \equiv V_m(\omega) f(\omega)$. If we then differentiate both sides with respect to ω we have:

$$(x'(\omega) - 1) \frac{\alpha + \beta(m(\omega) - \hat{m})}{\beta(m(\omega) - \hat{m})} - (x(\omega) - \omega) \frac{\alpha\beta m'(\omega)}{\beta^2(m(\omega) - \hat{m})^2} = h'(\omega). \tag{40}$$

Note

$$h'(\omega) = \frac{d}{d\omega}\left(\frac{\int_0^\omega g(\omega') \, d\omega'}{g(\omega)}\right) = 1 - \frac{g'(\omega)}{g(\omega)} h(\omega)$$

Furthermore,

$$\begin{aligned} \frac{g'(\omega)}{g(\omega)} &= \frac{V_m(\omega) f'(\omega)}{V_m(\omega) f(\omega)} + \frac{f(\omega)}{V_m(\omega) f(\omega)} \left(-\frac{m'(\omega)}{V(\omega)} + \frac{(m(\omega) - \hat{m}) V'(\omega)}{V(\omega)^2}\right) \\ &= \frac{f'(\omega)}{f(\omega)} - \frac{V(\omega)}{(m(\omega) - \hat{m})} \left(\frac{\beta(\omega + b - x) x'(\omega)}{V(\omega)} + \frac{\alpha(x(\omega) - \omega)(m(\omega) - \hat{m})}{V(\omega)^3}\right) \\ &= \frac{f'(\omega)}{f(\omega)} - \frac{\beta(\omega + b - x)}{(m(\omega) - \hat{m})} x'(\omega) - \frac{\alpha(x(\omega) - \omega)}{V(\omega)^2}, \end{aligned}$$

where the second line follows by the envelope theorem, which implies $V'(\omega) = \alpha(x - \omega)/V(\omega)$, and the IC condition, which says $m'(\omega) = -\beta(\omega + b - x)x'(\omega)$.

Rearranging equation (40) (and dropping the arguments of the functions for notational convenience) we have:

$$\begin{aligned} (x' - 1) \frac{\alpha + \beta(m - \hat{m})}{\beta(m - \hat{m})} + \frac{\alpha\beta \frac{(x - \omega)}{(m - \hat{m})}(\omega + b - x)}{\beta(m - \hat{m})} x' &= 1 - \left(\frac{f'}{f} - \frac{\alpha(x - \omega)}{V^2}\right) h + \frac{\beta(\omega + b - x) h}{(m - \hat{m})} x' \\ \left[\alpha + \beta(m - \hat{m}) + \alpha\beta \frac{(x - \omega)}{(m - \hat{m})}(\omega + b - x) - \beta^2(\omega + b - x) h\right] x'(\omega) &= \alpha + 2\beta(m - \hat{m}) - \left(\frac{f'}{f} - \frac{\alpha(x - \omega)}{V^2}\right) \beta(m - \hat{m}) h \end{aligned}$$

Now, by the definition of h we have:

$$\begin{aligned}\beta^2(\omega+b-x)h &= \beta^2(\omega+b-x)\,(x-\omega)\left(\frac{\alpha+\beta(m-\hat{m})}{\beta(m-\hat{m})}\right)-b\beta^2(\omega+b-x)\\ &= \alpha\beta\frac{(x-\omega)}{(m-\hat{m})}(\omega+b-x)-\beta^2(\omega+b-x)^2,\end{aligned}$$

so that the above becomes, after simplifying,

$$\begin{aligned}x'(\omega) &= \frac{\alpha+2\beta(m-\hat{m})+\left(\beta V_x V_m-\frac{f'}{f}\right)h}{\alpha+\beta(m-\hat{m})+\beta^2(\omega+b-x)^2}=\frac{n(\omega)}{d(\omega)}\\ &= \frac{\alpha+2\beta(m-\hat{m})+\left(\frac{\beta V_x}{f}-\frac{f'}{f^2V_m}\right)\int_0^\omega V_m(\omega')\,f(\omega')\ \mathrm{d}\omega'}{\alpha+\beta(m-\hat{m})+\beta^2(\omega+b-x)^2}.\end{aligned}$$

Note that a solution for x' always exists. To see this, note that a solution to our optimal control problem exists and since the above is a necessary condition for that solution it must be that it's satisfied at that solution. Thus the existence of a solution to the above is guaranteed. □

Proof of Theorem 4.10. Recall also that any solution in our optimal control problem satisfies: $x(\omega)\le\omega+b$ for all ω. Note that if $x(\omega)=\omega+b$ for some positive measure set of ω, then clearly for such ω we have $x'(\omega)=1>0$. Thus, in proving the existence of a fully separating solution, we take $x(\omega)<\omega+b$.

We will proceed through a number of lemmata before proving the main theorem.

Lemma B.2.4. *If $x(1)>1-b$, then $\alpha+2\beta(m(\omega)-\hat{m})>0$ for all $\omega\in\Omega$.*

Proof. By Lemma 4.1 and fact 1, $m(\cdot)$ is non-increasing. Thus, it suffices to show that $\alpha+2\beta(m(1)-\hat{m})>0$. From the FOC (10),

$$m(1)-\hat{m}=\frac{\alpha}{\beta}\left(\frac{x(1)-1}{1+b-x(1)}\right).$$

Thus

$$\alpha+2\beta(m(1)-\hat{m})=\alpha\left(\frac{x(1)-(1-b)}{1+b-x(1)}\right)>0.$$

□

Denote by $n(\omega)$ and $d(\omega)$ the numerator and denomenator in equation (14), that is, let $x'(\omega)=n(\omega)/d(\omega)$, where

$$n(\omega)\equiv\alpha+2\beta(m-\hat{m})+\beta V_x\int_0^\omega V_m(\omega')\ \mathrm{d}\omega'$$

and

$$d(\omega)\equiv\alpha+\beta(m-\hat{m})+\beta^2(\omega+b-x)^2.$$

Lemma B.2.5. *If $x(1) > 1-b$, the denominator of x' satisfies:*

$$d(\omega) = \alpha + \beta(m-\hat{m}) + \beta^2 (\omega + b - x)^2 > 0 \quad \textit{for all } \omega \in \Omega.$$

Proof. If $(m-\hat{m}) > 0$ then all of the terms in the denominator are positive. Otherwise, $(m-\hat{m}) < 0$ and $\alpha + \beta(m-\hat{m}) > \alpha + 2\beta(m-\hat{m}) > 0$ by lemma B.2.4. □

Thus, if $x(1) > 1-b$, the sign of $x'(\omega)$ is the same as the sign of $n(\omega)$. We next note that if $x(\omega) > \omega$ ($V_x < 0$), then $x' > 0$.

Lemma B.2.6. *Suppose $x(1) > 1-b$. If $x(\omega) > \omega$ (equivalently, $V_x < 0$), then:*

$$n(\omega) = \alpha + 2\beta(m-\hat{m}) + \beta V_x \int_0^{\omega} V_m(\omega')\, d\omega' > 0 \Rightarrow x' > 0.$$

Proof. We have shown before that $\int_0^{\omega} V_m(\omega') f(\omega')\, d\omega' \leq 0$ for all ω. By hypothesis, $V_x < 0$. By lemma B.2.4,if $x(1) > 1-b$, then $\alpha + 2\beta(m(\omega) - \hat{m}) > 0$ for all ω. Thus, all the terms in the numerator are positive. □

It follows that if $x'(\omega) \leq 0$ at some $\omega \in \Omega$ then $x(\omega) < \omega$. The following lemma is the key step in the proof of Theorem 4.10.

Lemma B.2.7. *Suppose $x(1) > 1-b$. If $x'(\omega) = 0$, then $x''(\omega) \leq 0$.*

Proof. When $x'(\omega) = 0$, then $n(\omega) = 0$ and $x''(\omega) = n'(\omega)\, d(\omega)$. Since lemma B.2.5 implies $d(\omega) > 0$, it suffices to show that $n'(\omega) \leq 0$.

Lemma B.2.6 proves that $x'(\omega) = 0$ implies $x(\omega) \leq \omega$, or $V_x \geq 0$. Furthermore, by the envelope theorem:

$$\begin{aligned} V_x'(\omega) &= \alpha \frac{(1 - x'(\omega))}{V(\omega)} + \frac{\alpha (x-\omega) V'(\omega)}{V(\omega)^2} \\ &= \alpha \frac{(1 - x'(\omega))}{V(\omega)} + \frac{\alpha^2 (x-\omega)^2}{V(\omega)^3} \\ &= \frac{1}{V}\left(\alpha (1-x') + V_x^2\right), \end{aligned}$$

or $V_x' = (\alpha + V_x^2)/V$ when $x' = 0$.

Now, the derivative of the numerator at $x' = 0$ is:

$$\begin{aligned} n'(\omega) &= 2\beta m'(\omega) + \beta V_x(\omega) V_m(\omega) + \beta V_x' \int_0^{\omega} V_m(\chi) f(\chi)\, d\chi \\ &= \beta V_x V_m + \frac{\beta}{V}\left(\alpha + V_x^2\right) \int_0^{\omega} V_m \\ &= \beta V_x V_m + \frac{\beta (\alpha + V_x^2)}{V} \left(\int_0^{\omega} V_m\right). \end{aligned}$$

Note that if $V_m \leq 0$, we have that $n'(\omega) \leq 0$, since $V_x \geq 0$ and $\int_0^\omega V_m \leq 0$ for all ω. Thus it suffices to consider the case where $V_m > 0$.

Multiply both sides by $V(\omega) > 0$ to get

$$n'(\omega)V = \beta V_x V_m V + \alpha\beta\left(\int_0^\omega V_m\right) + \beta V_x^2\left(\int_0^\omega V_m\right)$$

Since $\beta V_x \int_0^\omega V_m(\omega') f(\omega') d\omega' = -(\alpha + 2\beta(m - \hat{m}))$, this is

$$n'(\omega)V = \beta V_x V_m V + \alpha\beta\left(\int_0^\omega V_m\right) - V_x(\alpha + 2\beta(m - \hat{m}))$$

Now, from the $x(\omega)$-FOC,

$$\beta\int_0^\omega V_m = [V_x - V_m\beta(\omega + b - x(\omega))]$$

and thus

$$n'(\omega)V = \beta V_x V_m V - \alpha V_m \beta(\omega + b - x(\omega)) - 2\beta(m - \hat{m})V_x$$

Now, $V_x/V_m = \alpha(x - \omega)/(m - \hat{m})$, so $V_x = \alpha((x - \omega)/(m - \hat{m}))V_m$, and then

$$n'(\omega)V = \beta V_m[V_x V + \alpha(\omega - x - b)]$$

Finally, note that $V_x = \alpha(\omega - x)/V$, so

$$n'(\omega)V = \beta\alpha V_m(2(\omega - x) - b)$$

We want to find $\max_\omega(2\omega - 2x(\omega) - b)$. Note that this cannot be at an interior point since this would require $x' = 1$ and we have $x' = 0$. Thus it must be either when $\omega = x$ or when $\omega = 1$. If $\omega = x$, then $n'(\omega)V = -\alpha\beta V_m b \leq 0$, since $V_m > 0$. And if $\omega = 1$, then

$$n'(\omega)V = \alpha\beta V_m(2(1 - x(1)) - b) \leq 0,$$

where the last step follows by assumption 1, i.e., $x(1) \geq 1 - b/2$. □

We are finally ready to prove the theorem. Lemma 4.9 implies that if ever $x'(\widehat{\omega}) = 0$ for any $\widehat{\omega}$ then $x'(\omega) \leq 0$ for all $\omega \geq \widehat{\omega}$. Thus, if $x'(1) > 0$ we have that $x'(\omega) > 0$ for all ω and we have a separating solution. Now, by Lemma B.2.3,

$$x'(1) = \frac{\alpha + 2\beta(m(1) - \hat{m})}{\alpha + \beta(m(1) - \hat{m}) + \beta^2(1 + b - x(1))^2}$$

The denominator of $x'(1)$ is strictly positive by lemma B.2.5. Furthermore, since by Assumption 1 we have $x(1) > 1-b$, the numerator is also positive, since:

$$\begin{aligned}\alpha + 2\beta(m(1) - \hat{m}) &= \alpha\frac{(1+b-x(1))}{(1+b-x(1))} + \frac{2\alpha(x(1)-1)}{(1+b-x(1))} \\ &= \frac{\alpha(x(1)-1+b)}{1+b-x(1)} > 0.\end{aligned}$$

□

Proof of Lemma 4.11. We know that from the FOC at $\omega = 1$:

$$\frac{\alpha(x(1)-1)}{1+b-x(1)} = \beta(m(1) - \hat{m}),$$

and from the solution for m (and since $m_0 \geq \hat{m}$) we have that:

$$\begin{aligned}m(1) - \hat{m} &= \frac{\beta}{2}x(1)^2 - \beta(1+b)x(1) + \beta\int_0^\omega x(r)\ \mathrm{d}r + \frac{\beta}{2}x_0(2b - x_0) + m_0 - \hat{m} \\ &> -\beta(1+b)x(1).\end{aligned}$$

This inequality is not tight, but was chosen to give simple sufficient conditions. Thus:

$$\frac{\alpha(x(1)-1)}{1+b-x(1)} > -\beta^2(1+b)x(1)$$

Solving for $x(1)$, we get an inequality of the form:

$$x(1) > \frac{1+b}{2} + \frac{\alpha}{2\beta^2(1+b)} - \frac{1}{2}\sqrt{(1+b)^2 + \frac{\alpha(\alpha - 2\beta^2(1-b^2))}{\beta^4(1+b)^2}},$$

and to ensure that $x(1) \geq 1 - b/2$, it suffices that the right-hand side of the above inequality excees $1 - b/2$. This is always true if $b > 2$ (recall that prior to making approximations we knew we had a separating solution when $b > 1$) and when $b < 2$ we require that β is sufficiently small, in particular that:

$$\beta^2 \leq \frac{2\alpha}{6 + 3b - 3b^2}.$$

Since this condition is most binding when $b = 1/2$, we note that if $\beta^2 < 8\alpha/27$, then assumption 1 is also satisfied for all $b > 0$. □